53. *Theorems on the Finite-dimensionality of Cohomology Groups. III*

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The purpose of this note is to present some theorems on finite-dimensionality of cohomology groups attached to an elliptic system $\mathcal{M}$ of linear differential equations defined on an open manifold M with smooth boundary. We also give a theorem (Theorem 3), which may be regarded as a generalization of Martineau's duality theorem (Martineau [8]). The differential operators are always assumed to be of real analytic coefficients and the manifold under consideration is always assumed to be real analytic. We use the same notations as in our previous notes [4],[5] and do not repeat their definitions if there is no fear of confusions. The detailed arguments of this note shall be given somewhere else. The present writer expresses his heartiest thanks to Mr. M. Kashiwara for many valuable discussions concerning the theory of derived category.

The conditions on M and the system $\mathcal{M}$ imposed in Theorems 1 and 3 are the following:

(1) M is a relatively compact open submanifold of a (paracomspact) manifold L.

(2) The boundary N of M is non-singular, i.e. a real analytic submanifold of L.

(3) The system $\mathcal{M}$ is an admissible system defined on L, i.e. there exists locally a coherent left $\mathcal{D}_L^f$-Module $\mathcal{M}^f$ such that $\mathcal{M}=\mathcal{D}_L\underset{\mathcal{D}_L^f}{\otimes}\mathcal{M}^f$.

(4) The system $\mathcal{M}$ is elliptic on L, i.e. its characteristic variety V never intersects $\sqrt{-1}S^*L$.

(5) The system $\mathcal{M}$ admits a resolution of the following form:

$$0\leftarrow\mathcal{M}\leftarrow\mathcal{D}_L^{r_0}\leftarrow\mathcal{D}_L^{r_1}\leftarrow\cdots\leftarrow\mathcal{D}_L^{r_q}\leftarrow\mathcal{D}_L^{r_{q+1}}.$$

(In Theorem 3 we assume further that it has a free resolution of length d by $\mathcal{D}_L$.)

Before stating our theorems we prepare some notations related to the boundary value problem for an elliptic system of linear differential equations developed in Kashiwara-Kawai [2], [3]. Note that the codimension of N in L is 1 in our case and that this fact makes the situation very simple.