52. *On Dual Multiplicative Functionals*

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§1. Introduction and the results. Let X and $\hat{X}$ be standard (Markov) processes which are in duality relative to some measure. Let (M_t) be a multiplicative functional of X. R. K. Getoor [2] has proved the existence and uniqueness of a dual (exact) multiplicative functional $(\hat{M}_t)$ of (M_t) under the hypothesis of absolute continuity. The purpose of this note is to extend the results of Getoor to the general case without the above hypothesis.

We begin with some notation and terminology of Markov processes, following the book of Blumenthal-Getoor [1]. Let

$$X=(\Omega, \mathcal{F}, \mathcal{F}_t, X_t, \theta_t, P^x) \quad \text{and} \quad \hat{X}=(\hat{\Omega}, \hat{\mathcal{F}}, \hat{\mathcal{F}}_t, \hat{X}_t, \hat{\theta}_t, \hat{P}^x)$$

be standard processes with the same state space E. Their semi-groups and resolvents are, respectively, denoted by $(P_t), (\hat{P}_t), (U_\alpha)$ and $(\hat{U}_\alpha)$. We shall say that X and $\hat{X}$ are *in duality* relative to a Radon measure ξ, if

$$\int f(x)U_\alpha g(x)\xi(dx)=\int \hat{U}_\alpha f(x)g(x)\xi(dx) \qquad \alpha>0$$

or equivalently

$$\int f(x)P_t g(x)\xi(dx)=\int \hat{P}_t f(x)g(x)\xi(dx) \qquad t\geqslant 0$$

for any nonnegative and universally measurable functions f and g. We *do not assume* the resolvents (U_α) and $(\hat{U}_\alpha)$ are absolutely continuous relative to ξ. In the following the integral $\int f(x)g(x)\xi(dx)$ is written as (f, g). As usual $\mathcal{E}$ (resp. $\mathcal{E}^*$) is a σ-algebra of Borel (resp. universally measurable) subsets of E, and $f\in\mathcal{E}$ (resp. $\mathcal{E}^*$) means that f is $\mathcal{E}$ (resp. $\mathcal{E}^*$)-measurable.

Let (M_t) be a multiplicative functional (abbreviated as MF) of X. In this paper all MF's are assumed to be right continuous, decreasing, and to satisfy $0\leqslant M_t\leqslant 1$. The semigroup and resolvent generated by (M_t) are denoted by (Q_t) and (V_α):

$$Q_t f(x)=E^x[f(X_t)M_t], \qquad t\geqslant 0,$$

$$V_\alpha f(x)=E^x\int_0^\infty e^{-\alpha t}f(X_t)M_t dt, \qquad \alpha>0.$$

A $MF(\hat{M}_t)$ of $\hat{X}$ is said to be a *dual multiplicative functional* of (M_t) if the relation

$$(f, V_\alpha g)=(\hat{V}_\alpha f, g) \qquad \alpha>0$$