68. *A Note on the Asymptotic Behavior of the Solutions of* $\dddot{x}+a(t)f(\ddot{x})\dddot{x}+b(t)\phi(\dot{x},\ddot{x})+c(t)g(\dot{x})+d(t)h(x)=p(t,x,\dot{x},\ddot{x},\dddot{x})$

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1. Introduction. In this note we shall be concerned with fourth order non-autonomous differential equations of the form

(1.1) $\ddddot{x}+a(t)f(\ddot{x})\dddot{x}+b(t)\phi(\dot{x},\ddot{x})+c(t)g(\dot{x})+d(t)h(x)=p(t,x,\dot{x},\ddot{x},\dddot{x})$

where $a, b, c, d, f, \phi, g, h$ and p are continuous real-valued functions depending only on the arguments displayed, and dots indicate differentiation with respect to t.

Many authors (J. O. C. Ezeilo [3], M. Harrow [6], A. S. C. Sinha and R. G. Hoft [10], M. A. Asmussen [1], B. S. Lalli and W. S. Skrapek [8], T. Hara [4], etc. [9]) have studied the stability of the trivial solution of the fourth order autonomous differential equations of the form

(1.2) $$\ddddot{x}+f(\ddot{x})\dddot{x}+\phi(\dot{x},\ddot{x})+g(\dot{x})+h(x)=0$$

and their perturbed equations of the form

(1.3) $$\ddddot{x}+f(\ddot{x})\dddot{x}+\phi(\dot{x},\ddot{x})+g(\dot{x})+h(x)=p(t,x,\dot{x},\ddot{x},\dddot{x}).$$

We shall investigate sufficient conditions under which all solutions of the non-autonomous differential equation (1.1) tend to zero as $t\to\infty$.

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2. Assumptions and theorem. Let us begin by stating the assumptions on the functions appeared in the equation (1.1).

Assumptions.

(I) $a(t), b(t), c(t)$ and $d(t)$ are C^1-functions in $I=[0,\infty)$.

(II) $f(z)$ is a C^1-function in R^1.

(III) The functions $\phi(y,z)$ and $\frac{\partial\phi}{\partial y}(y,z)$ are continuous in R^2.

(IV) $g(y)$ is a C^1-function in R^1.

(V) $h(x)$ is a C^1-function in R^1.

(VI) The function $p(t,x,y,z,w)$ is continuous in $I\times R^4$.

Hereafter the following notations are used:

$$g_1(y)=\frac{g(y)}{y}\quad(y\neq 0),\qquad g_1(0)=g'(0),$$

$$f_1(z)=\frac{1}{z}\int_0^z f(\zeta)d\zeta\quad(z\neq 0),\qquad f_1(0)=f(0).$$

Our result is summarized in the following theorem: