98. *Expandability and Product Spaces*

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1. Introduction. Let $\mathfrak{m}$ be an infinite cardinal number. A topological space X is said to be *$\mathfrak{m}$-expandable* (resp. *discretely $\mathfrak{m}$-expandable*), if for every locally finite (resp. discrete) collection $\{F_\lambda \mid \lambda \in \Lambda\}$ of subsets of X with $|\Lambda| \leq \mathfrak{m}$, where $|\Lambda|$ denotes the power of Λ, there exists a locally finite collection $\{G_\lambda \mid \lambda \in \Lambda\}$ of open subsets of X such that $F_\lambda \subset G_\lambda$ for every $\lambda \in \Lambda$. A collection $\{G_\lambda \mid \lambda \in \Lambda\}$ of subsets of a topological space is said to be *hereditarily conservative* (H.C.) if every collection $\{H_\lambda \mid \lambda \in \Lambda\}$, such that $H_\lambda \subset G_\lambda$ for every $\lambda \in \Lambda$, is closure preserving. A topological space X is said to be *H.C. $\mathfrak{m}$-expandable* (resp. *discretely H.C. $\mathfrak{m}$-expandable*), if for every locally finite (resp. discrete) collection $\{F_\lambda \mid \lambda \in \Lambda\}$ of subsets of X with $|\Lambda| \leq \mathfrak{m}$, there exists a hereditarily conservative collection $\{G_\lambda \mid \lambda \in \Lambda\}$ of open subsets of X such that $F_\lambda \subset G_\lambda$ for every $\lambda \in \Lambda$. A topological space is said to be *expandable, discretely expandable, H.C. expandable* or *discretely H.C. expandable*, respectively, if it is $\mathfrak{m}$-expandable, discretely $\mathfrak{m}$-expandable, H.C. $\mathfrak{m}$-expandable or discretely H.C. $\mathfrak{m}$-expandable for every cardinal number $\mathfrak{m}$ ([1], [2]).

In [1] and [2], Krajewski and Smith showed the following:

(i) X is $\aleph_0$-expandable if and only if X is countably paracompact.

(ii) X is $\mathfrak{m}$-expandable if and only if X is discretely $\mathfrak{m}$-expandable and countably paracompact.

(iii) X is collectionwise normal if and only if X is discretely expandable and normal.

Let $T(\mathfrak{m})$ be a set whose power is $\mathfrak{m}$ and t_0 be a distinguished element of $T(\mathfrak{m})$. On $T(\mathfrak{m})$ we define a topology by the following: A subset of $T(\mathfrak{m})$ is open if and only if it dose not contain t_0 or its complement is finite. Then $T(\mathfrak{m})$ is a compact Hausdorff space. If X is a topological space, then in the product space $X \times T(\mathfrak{m})$ let $X_0 = X \times \{t_0\}$.

The main purpose of this paper is to show the following theorem which is a generalization of Martin [3, Lemma 1].

Theorem 1. *The following statements are equivalent for a topological space X.*

(a) *X is $\mathfrak{m}$-expandable.*

(b) *$X \times T(\mathfrak{m})$ is $\mathfrak{m}$-expandable.*