96. *Cyclotomic Algebras over a 2-adic Field*

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(Comm. by Kenjiro SHODA, M. J. A., June 12, 1973)

1. Let K be a finite extension of Q_2, the rational 2-adic numbers. E. Witt [5] proved that the order of the Schur subgroup $S(K)$ of the Brauer group $Br(K)$ is 1 or 2. So, given any finite extension K of Q_2, we must tell whether $S(K)=1$ or $S(K)$ is the subgroup of $Br(K)$ of order 2. This problem was completely settled by the author [3]. The purpose of the present paper is to outline another proof of the result. (The details will appear in the lecture note [4].) The idea of the new proof is the same as the one devised by the author in [1], where for any finite extension K of the rational p-adic numbers Q_p, p being any odd prime, the Schur subgroup $S(K)$ was determined.

Notation. For a positive integer n, ζ_n is a primitive nth root of unity. Let $L \supset k$ be extensions of Q_p such that L/k is normal. Then $G(L/k)$ is the Galois group of L over k. $e_{L/k}$ (resp. $f_{L/k}$) denotes the ramification index (resp. the residue class degree) of L/k.

2. Throughout this paper, k denotes a cyclotomic extension of Q_2. Let B be a *cyclotomic algebra* over k:

$$B=(\beta, k(\zeta)/k)=\sum_{\sigma\in G} k(\zeta)u_\sigma \text{ (direct sum)}, \qquad (u_1=1),$$

$$u_\sigma u_\tau=\beta(\sigma,\tau)u_{\sigma\tau}, \quad u_\sigma x = x^\sigma u_\sigma \qquad (x\in k(\zeta)),$$

where ζ is a root of unity, $G=G(k(\zeta)/k)$, and β is a factor set of $k(\zeta)/k$ such that the values of β are roots of unity in $k(\zeta)$. Let $L=Q_2(\zeta')$ be a cyclotomic field containing $k(\zeta)$, ζ' being some root of unity. Let Inf denote the inflation map from $H^2(k(\zeta)/k)$ into $H^2(L/k)$. Then $B\sim(\mathrm{Inf}(\beta), L/k)$. Thus we always assume that any cyclotomic algebra B over k is of the form: $B=(\beta, L/k)$, L being a cyclotomic field over Q_2. We can write $L=Q_2(\zeta_{2^n}, \zeta_r)$, $r=2^a-1$, where $a=f_{L/Q_2}$ and n is some non-negative integer. If $n\leq 1$, then $B\sim 1$, because the extension L/k is unramified and the factor set β consists of roots of unity. So we assume $n\geq 2$. We have $\beta(\sigma,\tau)=\alpha(\sigma,\tau)\gamma(\sigma,\tau)$, $\alpha(\sigma,\tau)\in\langle\zeta_{2^n}\rangle$, $\gamma(\sigma,\tau)\in\langle\zeta_r\rangle$, for any σ, τ of $G(L/k)$, whence $(\beta, L/k)\sim(\alpha, L/k)\otimes_k(\gamma, L/k)$.

Proposition 1 (Witt [5, pp. 242–243]). $(\gamma, L/k)\sim 1$.

Remark. The result can also be proved by the techniques that will be developed in this paper. (See [4].) Another proof was already given in [3].

Thus we only need to study the following type of cyclotomic