91. *On the Channel Capacity of a State Machine**)

By Yatsuka NAKAMURA
Faculty of Engineering, the Shinshu University, Nagano

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1. Introduction. In the information theory, the calculation of the channel capacity is not easy in general. The calculation method is known for only memoryless channels and several particular cases.

Finite automata are seen as information channels in various way (for example, see [1]). A kind of such automata, called a permutation machine, is described as a stationary 0-memory Markovian channel ([7]). In this paper, we shall give the method to get the capacity of such channels.

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2. Permutation channel. Let $\mathcal{A}=\{S, X, \tau\}$ be a state machine, i.e., (i) S is a non-empty finite set of states, (ii) X is a non-empty finite set of input letters, and (iii) τ is a mapping from $S\times X$ to S, called a transition function. A state machine can be represented by a finite directed graph G, where states correspond to vertices and transitions to directed edges indexed by elements in X. Terminology of the graph theory used here refers to Ore [5]. A directed edge (s_1, s_2) indexed by x is denoted by $s_1\xrightarrow{x}s_2$, which exists if and only if $\tau(s_1, x)=s_2$. For such graph, let us assume the following property: (A) For every vertex s and every input letter x, there exists one and only one directed edge of the form $s_1\xrightarrow{x}s$ for some state s_1, i.e., for every input letter x, a mapping $\tau(\cdot, x)$, which is from S onto S, is a permutation on S. A state machine, a graph of which satisfies the condition (A), is called a permutation machine (cf. [3] p. 195).

Let $X^I(I=\{0, \pm 1, \pm 2, \cdots\})$ be an alphabet space, where the state space X is a set of input letters of a permutation machine. And S^I be another alphabet space, where S is a set of states of the machine. For any sequence $s_is_{i+1}\cdots s_{i+l}$ of states in S, we define an information channel ν by

$$\nu_{\bar{x}}(s_is_{i+1}\cdots s_{i+l})=\frac{1}{N}q^{x_{i+1}}_{s_is_{i+1}}q^{x_{i+2}}_{s_{i+1}s_{i+2}}\cdots q^{x_{i+l}}_{s_{i+l-1}s_{i+l}} \qquad (\bar{x}\in X^I)$$

where

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