90. *On Normal Approximate Spectrum.* V

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1. Introduction. In our previous notes [4]–[7] and [9], we have discussed some properties of the normal approximate spectra of operators on a Hilbert space $\mathfrak{H}$.

A complex number λ is an *approximate propervalue* of an operator T on $\mathfrak{H}$ if there is a sequence $\{x_n\}$ of unit vectors in $\mathfrak{H}$ such that

$$(*) \qquad \|(T-\lambda)x_n\| \to 0 \qquad (n\to\infty).$$

$\{x_n\}$ is called a normal approximate propervectors belonging to λ. The set $\pi(T)$ of all approximate propervalues is called the *approximate spectrum* of T. If $\{x_n\}$ satisfies $(*)$ and

$$(**) \qquad \|(T-\lambda)^* x_n\| \to 0 \qquad (n\to\infty),$$

then λ is called a *normal approximate propervalue* of T and $\{x_n\}$ normal approximate propervectors belonging to λ. The set $\pi_n(T)$ of all normal approximate propervalues of T is called the *normal approximate spectrum* of T.

Bunce [2] initiated to discuss the mutual dependency among the approximate propervalues of an operator T and the characters of the unital C^*-algebra $\mathfrak{A}$ generated by T. He established, among others, the reciprocity for hyponormal operators. The reciprocity for general operators is obtained in [4] and [9]. In the present note, we shall give an alternative proof of the reciprocity basing on the Berberian representation of an operator established by Berberian [1]:

Theorem A (Berberian). *For a Hilbert space $\mathfrak{H}$, there is a Hilbert space $\mathfrak{K}$ such that*

(i) *an operator T acting on $\mathfrak{H}$ is represented by an operator T^0 acting on $\mathfrak{K}$ which satisfies*

$$(1) \qquad \pi(T) = \pi(T^0) = \sigma_p(T^0)$$

where $\sigma_p(T^0)$ is the point spectrum of T^0, and

(ii) *the Berberian representation: $T \to T^0$ is $*$-isomorphic and isometric.*

In the remainder of the present note, we shall give another proofs of theorems of Hildebrandt [8] and Bunce [3] also basing on the Berberian representation.

2. Reciprocity. Let $\mathfrak{A}$ be the C^*-algebra generated by an operator T and the identity. By a *character* of $\mathfrak{A}$ we mean a multiplicative