89. On Normal Approximate Spectrum. IV

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(Comm. by Kinjirô KUNUGI, M. J. A., June 12, 1973)

1. Introduction. In our previous notes [3], [5], [6] and [7], we have discussed some properties of the normal approximate spectra of operators on Hilbert space $\mathfrak{H}$. A complex number λ is an *approximate propervalue* of an operator T on $\mathfrak{H}$ if there is a sequence of unit vectors in $\mathfrak{H}$ such that

$$(*) \qquad \|(T-\lambda)x_n\|\to 0 \qquad (n\to\infty).$$

Then sequence $\{x_n\}$ is called *approximate propervectors* belonging to λ. The set $\pi(T)$ of all approximate propervalues is called the *approximate spectrum* of T. If there is a sequence $\{x_n\}$ of unit vectors for λ and T satisfying $(*)$ and

$$(**) \qquad \|(T-\lambda)^*x_n\|\to 0 \qquad (n\to\infty),$$

the λ is called a *normal approximate propervalue* of T and $\{x_n\}$ *normal approximate propervectors*. The set $\pi_n(T)$ of all normal approximate propervalues of T is called the *normal approximate spectrum* of T. Some equivalent conditions are discussed in [3], [5] and [7].

In the present note, we shall prove three theorems in terms of the normal approximate spectra in §§ 3–5. In the proofs, we shall use the Berberian representation in [1], which is sketched in § 2.

2. The Berberian representation. Let $\mathfrak{B}$ be the set of all bounded sequences of vectors of $\mathfrak{H}$. Then $\mathfrak{B}$ is a vector space with respect to the operations:

$$\{x_n\}+\{y_n\}=\{x_n+y_n\}$$

and

$$\alpha\{x_n\}=\{\alpha x_n\}.$$

Let (for a fixed Banach limit Lim)

$$\mathfrak{N}=\{\{x_n\}\in\mathfrak{B}\,;\ \operatorname*{Lim}_{n\to\infty}(x_n|y_n)=0 \text{ for all } y_n\in\mathfrak{B}\},$$

and let $\mathfrak{P}=\mathfrak{B}/\mathfrak{N}$. Then $\mathfrak{P}$ becomes an inner product space by

$$(\{x_n\}+\mathfrak{N}\,|\,\{y_n\}+\mathfrak{N})=\operatorname*{Lim}_{n\to\infty}(x_n|y_n).$$

If $x\in\mathfrak{H}$, then $\{x\}$ means the sequence of all whose terms are x.

$$(x'|y')=(x|y)$$

for $x'=\{x\}+\mathfrak{N}$ and $y'=\{y\}+\mathfrak{N}$, so that the mapping $x\to x'$ is an isometric linear map of $\mathfrak{H}$ onto a closed subspace $\mathfrak{H}'$ of $\mathfrak{P}$. Let $\mathfrak{K}$ be the

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