# 87. *Cotangential Decomposition of the Sheaf $\mathcal{D}'/\mathcal{E}$*

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(Comm. by Kôsaku YOSIDA, M. J. A., June 12, 1973)

The aim of this note is to construct a sheaf in the distribution theory which has analogous properties to those of the sheaf $\mathcal{C}$ important in the hyperfunction theory.

Let $\Omega$ be a domain in $\boldsymbol{R}^n$ and let $\mathcal{D}'$, $\mathcal{E}$, $\mathcal{B}$ and $\mathcal{A}$ denote the sheaves of the germs of distributions, infinitely differentiable functions, hyperfunctions and real analytic functions in $\Omega$ respectively. The quotient sheaves $\mathcal{D}'/\mathcal{E}$, $\mathcal{B}/\mathcal{A}$ and $\mathcal{D}'/\mathcal{A}$ should be called the sheaves of singularities over $\Omega$. In 1969 M. Sato decomposed the sheaf $\mathcal{B}/\mathcal{A}$ into the cotangential directions. That is, he constructed a sheaf $\mathcal{C}$ over the cosphere bundle $S^*\Omega$ whose direct image $\pi_*\mathcal{C}$ along the projection $\pi$ onto the base space $\Omega$ is isomorphic to the sheaf $\mathcal{B}/\mathcal{A}$. Actually this induces an isomorphism of global sections:

$$\mathcal{B}(\Omega)/\mathcal{A}(\Omega)\cong\pi_*\mathcal{C}(\Omega)\cong\mathcal{C}(S^*\Omega).$$

The sheaf $\mathcal{C}$ is flabby as well as the sheaf $\mathcal{B}$. (See Sato-Kashiwara [3], Sato-Kawai-Kashiwara [4].)

Let $\mathcal{H}^s_{\mathrm{loc}}$ be the sheaf of distributions in the local Sobolev space $H^s_{\mathrm{loc}}(\Omega)$. In this note we decompose the sheaf $\mathcal{H}^s_{\mathrm{loc}}/\mathcal{E}$ to obtain a sheaf $\mathcal{M}^s$ over the cosphere bundle $S^*\Omega$ such that the following isomorphisms

$$H^s_{\mathrm{loc}}(\Omega)/\mathcal{E}(\Omega)\cong\pi_*\mathcal{M}^s(\Omega)\cong\mathcal{M}^s(S^*\Omega)$$

hold. This sheaf $\mathcal{M}^s$ is soft.

The supports of sections of $\mathcal{M}^s$ are closed subsets of the cosphere bundle $S^*\Omega$. These correspond to what is called "singular supports $S-S$" in the theory of the sheaf $\mathcal{C}$. Their projections to the base space $\Omega$ coincide with the classical singular supports of distributions. Our definition of the sheaf $\mathcal{M}^s$ is essentially the same as announced in Hörmander's paper [1]. And the wave front sets introduced by him in the case of $\mathcal{D}'/\mathcal{E}$ are nothing but the supports of the sections of our sheaf $\mathcal{M}^{-\infty}$.

Let $\omega$ be an open set in $\Omega$ and $\sigma$ be an open set in the unit sphere $S^{n-1}$ in $\boldsymbol{R}^n$.

We shall introduce linear spaces as the following.

$H^{s,\infty}_{\mathrm{loc}}(\omega\times\sigma)=\{u\in H^s_{\mathrm{loc}}(\omega)$; for any compact sets $K\subset\omega\subset\Omega$ and $\kappa\subset\sigma\subset S^{n-1}$, there exists a function $\phi_K\in C_0^\infty(\omega)$ such that (i) $\phi_K\geqq 0$ and $\phi_K\equiv 1$ near $K$ and (ii) for any positive integer $N$, $|\widehat{\phi u}(\xi)|\leqq C_N/(1+|\xi|)^N$ so long