84. On Infinitesimal Automorphisms and Homogeneous Siegel Domains over Circular Cones

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Let $D(V, F)$ be a homogeneous Siegel domain of type I or type II, where V is a convex cone in a real vector space R and F is a V-hermitian form on a complex vector space W. Let $C(n)$ be the *circular cone* of dimension n ($n\geq 3$), that is, the set $\{(x_1, \cdots, x_n)\in \boldsymbol{R}^n ; x_1>0, x_1x_2-x_3^2-, \cdots, -x_n^2>0\}$. In this note we will state a result on infinitesimal automorphisms of $D(V, F)$ and a method of constructing all homogeneous Siegel domains over circular cones. As an application, we will give the explicit form of a Siegel domain which is isomorphic to the exceptional bounded symmetric domain in $\boldsymbol{C}^{16}$ (; no explicit description of this Siegel domain has ever been obtained, as far as we know). The detailed results with their complete proofs will appear elsewhere.

1. Let $\mathfrak{g}_h$ (resp. $\mathfrak{g}_a$) denote the Lie algebra of all infinitesimal holomorphic (resp. affine) automorphisms of $D(V, F)$. Let $(z_1, \cdots, z_n, w_1, \cdots, w_m)$ be a canonical complex coordinate system of $R^C\times W$, where R^C is the complexification of R, $n=\dim_C R^C$, $m=\dim_C W$ and put $\partial=\sum_{1\leq k\leq n} z_k\partial/\partial z_k+1/2\sum_{1\leq \alpha\leq m} w_\alpha\partial/\partial w_\alpha$. Then the following results are known in [5], [10].

(1) $\mathfrak{g}_h=\mathfrak{g}_{-1}+\mathfrak{g}_{-1/2}+\mathfrak{g}_0+\mathfrak{g}_{1/2}+\mathfrak{g}_1$ *is a graded Lie algebra and* $\mathfrak{g}_a=\mathfrak{g}_{-1}+\mathfrak{g}_{-1/2}+\mathfrak{g}_0$, *where* $\mathfrak{g}_\lambda$ ($\lambda=0, \pm 1/2, \pm 1$) *is the λ-eigenspace of* $\mathrm{ad}(\partial)$. *Furthermore* $\mathfrak{g}_{-1}$ *is identified with R as vector spaces.*

Considering (1) we denote by ρ the adjoint representation of the subalgebra $\mathfrak{g}_0$ on $\mathfrak{g}_{-1}=R$, and we know $\rho(\mathfrak{g}_0)\subset\mathfrak{g}(V)\subset\mathfrak{gl}(R)$, where $\mathfrak{g}(V)$ denotes the Lie algebra of $\mathrm{Aut}(V)=\{g\in GL(R) ; g(V)=V\}$. Using the descriptions of $\mathfrak{g}_{1/2}$, $\mathfrak{g}_1$ in terms of polynomial vector fields [7] and using the structure of the radical of $\mathfrak{g}_h$ [5] and the criterion of irreducibility of $D(V, F)$ [2], we get

Theorem 1. *If ρ is irreducible, then $\mathfrak{g}_h$ is simple or $\mathfrak{g}_h=\mathfrak{g}_a$.*

A homogeneous Siegel domain $D(V, F)$ of type II is said to be *non-degenerate* if the linear closure of $\{F(u, u) ; u\in W\}$ in R coincides with R (cf. [3]).

Remark. Without the assumption of irreducibility of ρ, we can