82. A Table of Hecke Operators. II

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Let q be a prime number such that $q\equiv 1 \pmod 4$ and $\Gamma_0(q)$ be the congruence subgroup of level q i.e.,

$$\Gamma_0(q)=\left\{\begin{pmatrix} a & b \\ c & d\end{pmatrix} \in SL_2(\boldsymbol{Z})\,;\ c\equiv 0 \pmod q\right\}.$$

Let $S(q)$ be the set of cusp forms $f(z)$ such that

$$f\left(\frac{az+b}{cz+d}\right)=(cz+d)^2 f(z), \text{ for all } \begin{pmatrix} a & b \\ c & d\end{pmatrix} \in \Gamma_0(q).$$

Then $S(q)$ forms a finite dimensional vector space over the complex number field. Let p be a prime number different from q. The Heck operator $T(p)$ is a linear transformation of $S(q)$ and it is known that if we choose suitable basis of $S(q)$ (independently of p), each $T(p)$ can be represented in the following form

$$T(p)=\begin{pmatrix} x_1 & & & 0 \\ & x_2 & & \\ & & \ddots & \\ 0 & & & x_n \end{pmatrix}$$

We can compute $x_1, x_2, \cdots, x_n$ using the Eichler-Selberg trace formula (cf. [1], [2]).

In general the characteristic polynomial of $T(p)$

$$F_{p,q}(x)=(x-x_1)(x-x_2)\cdots(x-x_n) \in \boldsymbol{Z}[x]$$

is not irreducible. But we have the following factorization algorithm: Suitable combinations of elementary symmetric functions in some of these roots $x_1, x_2, \cdots, x_n$ are tested in order to decide whether or not they are sufficiently close to rational integers to guarantee the existence of a corresponding proper factor of $F_{p,q}(x)$ in $\boldsymbol{Z}[x]$ (cf. [3]).

The author made a table of factorized $F_{p,q}(x)$ (with irreducible factors) for

(1) $\quad 0<q<250,\ q\neq 227, 239,$

(2) $\quad 0<p<1000,\ p\neq q.$

For computing this table, the author used the electronic computer TOSBAC-3000 installed in the Department of Mathematics, Tsuda College. This calculation required about three hundred hours computer time.

This table is very large. So in this paper there is only a list of