# 116. *Thin Sets in an Open Unit Disk*

By Masayuki OSADA
Department of Mathematics, Hokkaido University

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**1. Introduction.** The purpose of this paper is to establish the following theorem.

**Theorem.** *Let $F$ be a closed subset of an open unit disk $U=\{|z|<1\}$. Suppose the circular projection $T(F)$ of $F$ contains some countable union $\{E_n\}_{n=1}^{\infty}$ of closed intervals such that each $E_n$ $(n=1,2,\cdots)$ is a closed interval $[a_n, b_n]$ with $0<a_n<b_n<a_{n+1}<1$ and $\lim_{n\to\infty} a_n=1$. Set $\lambda_k=\inf_{x\in E_k}\sup_{z\in F,|z|=x} k_1(z)$ $(k=1,2,\cdots)$. If $\overline{\lim_{n\to\infty}}\frac{1}{1-a_n}\sum_{k=n}^{\infty}\lambda_k(b_k-a_k)(1-a_kb_k)>0$, then $F$ is not thin at $z=1$.*

**Notation and terminology.** Let $C$ be a complex plane. For a subset $A$ of $C$, we denote by $\partial A$ the boundary of $A$ in $C$.
Let $U$ be an open unit disk $\{|z|<1\}$ in $C$ in this paper. Set $T(z)=|z|$ $(z\in U)$. Then $T$ is a continuous mapping of $U$ into $U$. For a subset $A$ of $U$, we say that $T(A)$ is the circular projection of $A$. Let $a$ and $b$ two points of $U$. Then we define the hyperbolic distance (or length) $\delta(a, b)$ of $a$ and $b$ by $\delta(a,b)=\left|\frac{a-b}{1-\bar{a}b}\right|$. For a subset $A$ of $U$, the hyperbolic diameter $\delta(A)$ of $A$ is defined by $\delta(A)=\sup_{a,b\in A}\delta(a,b)$.

We shall use the same notations as in [3], for instance, $C_0(X)$, $\overline{H}_f^G$, $\underline{H}_f^G$, $H_f^G$, $\omega_a^G=\omega_a=\omega$, $s_F$, the Green capacity $C$, etc.

**2. Green potentials on $U$.** Let $\mu$ be a (positive Radon) measure on $U$. Set $L(f)=\int f\circ T d\mu$ for each $f$ of $C_0(U)$. Then $L$ is a positive linear functional on $C_0(U)$. By Riesz representation theorem, there exists a (positive Radon) measure $\mu^T$ on $U$ such that $L(f)=\int f d\mu^T$.

The following properties are easy to see:

(i) $\int f d\mu^T=\int f(|z|)d\mu(z)$ for any non-negative Borel measurable function $f$ on $U$,

(ii) $\int d\mu=\int d\mu^T$,

(iii) $S(\mu^T)=T(S_\mu)$, where $S_\mu$ is the support of $\mu$.

Let $g(z,\zeta)=\log\left|\frac{1-\bar{z}\zeta}{z-\zeta}\right|$ denote the Green function on $U$ with pole at $\zeta\in U$ and $p^\mu$ be a Green potential associated with a (positive Radon)