114. *Uniqueness of the Solution of Some Characteristic Cauchy Problems for First Order Systems*

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1. Introduction and definition. In this note we treat the following system;

$$A\frac{\partial u}{\partial t}=\sum_{j=1}^{n} B_j\frac{\partial u}{\partial x_j}+Cu, \tag{1.1}$$

where A, B_j $(j=1, \cdots, n)$ and C are all $N\times N$ matrices, and $u=u(t, x)$ is an N-vector. We consider the Cauchy problem for (1.1) with initial data on the hypersurface $t=0$. We concern here only with real analytic solutions, so we assume all the coefficients are real analytic in a neighborhood of $(t, x)=(0, 0)$. If A is regular in a neighborhood of $(t, x)=(0, 0)$, then we can find a unique solution for any analytic data by the well-known theorem of Cauchy-Kowalevskaya, however when A is singular at $t=0$ or in a neighborhood of $t=0$, many complicated affairs appear as for the existence or the uniqueness of the solution.

The case when A becomes singular only at $t=0$ was treated by M. Miyake [3], and he was concerned with the existence of the solution. For the single equation, one can refer to Y. Hasegawa [2].

The case which we treat here is that A is singular in a neighborhood of $(t, x)=(0, 0)$, and we consider the uniqueness of the solution. In our case the uniqueness of the solution is deeply related to the lower order term, that is, to the matrix C.

To classify our equation, we give the following definition.

Definition 1.1. The equation (1.1) is said to be of type (p, q), if and only if the rank of A is p and the degree of $F(\tau\,; t, x)$ as a polynomial in τ is q in a neighborhood of $(t, x)=(0, 0)$, where $F(\tau\,; t, x)$ denotes $det(\tau A-C)$.

Of course q does not exceed p, and if $p=N$, it is noncharacteristic case.

2. The case of constant coefficients. For the case of constant coefficients, we can obtain a necessary and sufficient condition for the solution of the Cauchy problem for (1.1) to be unique, if it is of type (p, p). Before stating the result, we refer to the following theorem which suggests the relation between the uniqueness of the solution and the lower order term.