113. *A Note on Cauchy Problems of Semi-linear Equations and Semi-groups in Banach Spaces*

By Yoshikazu KOBAYASHI

Department of Mathematics, Waseda University, Tokyo

(Comm. by Kinjirô KUNUGI, M. J. A., July 12, 1973)

§1. Introduction. Let X be a real Banach space with the norm $\| \ \|$. An operator B in X is said to be *accretive* if

(1.1) $\|(I+\lambda B)x-(I+\lambda B)y\|\geq\|x-y\|$ for $x, y \in D(B)$ and $\lambda>0$.

It is known that B is accretive if and only if for any $x, y \in D(B)$ there exists $f \in F(x-y)$ such that $(Bx-By, f)\geq 0$, where F is the duality map of X, i.e., $F(x)=\{x^* \in X^* ; (x, x^*)=\|x\|^2=\|x^*\|^2\}$ for $x \in X$. If B is accretive and $R(I+\lambda B)=X$ for all $\lambda>0$, we say that B is *m-accretive.*

Let A be a linear m-accretive operator in X with dense domain and let B be a nonlinear accretive operator in X. Recently G. Webb [4] proved that, under some additional assumptions on A and B, for all $x \in X$ and $t\geq 0$

(1.2) $$U(t)x=\lim_{n\to\infty}((I+(t/n)B)^{-1}(I+(t/n)A)^{-1})^n x$$

exists and $\{U(t) ; t\geq 0\}$ is a contraction semi-group on X. By a *contraction semi-group* on C, where C is a subset of X, we mean a family $\{U(t) ; t\geq 0\}$ of operators $U(t): C\to C$ satisfying the following conditions: (1) $U(t)U(s)=U(t+s)$ for $t, s\geq 0$; (2) $\lim_{t\to 0+}U(t)x=U(0)x=x$ for $x \in C$; (3) $U(t)$, $t\geq 0$, are contractions on C, i.e., $\|U(t)x-U(t)y\|\leq\|x-y\|$ for $x, y \in C$, $t\geq 0$.

In this paper, we shall study how the semi-group $\{U(t) ; t\geq 0\}$ given by (1.2) is related to the strong solution of the following Cauchy problem

(1.3) $$du/dt+(A+B)u=0, \qquad u(0)=x \ (\in X).$$

Now we give the precise definition of strong solution of the Cauchy problem (1.3).

Definition 1.1. A function $u: [0, \infty)\to X$ is a *strong solution* of (1.3) if u is Lipschitz continuous on $[0, \infty)$, $u(0)=x$, u is strongly differentiable almost everywhere and

(1.4) $$du(t)/dt+(A+B)u(t)=0 \qquad \text{for a.a. } t \in [0, \infty).$$

It follows easily from the accretiveness of $A+B$ that the Cauchy problem has at most one strong solution.

Our results are stated as follows; and the proofs are given in §2.

Theorem 1.1. *Suppose that A is a linear m-accretive operator in X with dense domain, B is a nonlinear accretive operator in X and D is a subset of $D(A)\cap D(B)$ satisfying $(I+\lambda B)^{-1}(I+\lambda A)^{-1}(D)\subset D$ for $\lambda>0$. Let $u: [0, \infty)\to X$ be a strong solution of the Cauchy problem* (1.3) *with*