# 110. On Nonexistence of Global Solutions of Some Semilinear Parabolic Differential Equations

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The purpose of this paper is to show that *the semilinear parabolic equation* $(\partial/\partial t)u=\Delta u+u^{1+\alpha}$ *has no global solutions for any nontrivial nonnegative initial data* $u_0(x)$ *in case of* $N=2$, $\alpha=1$ *or* $N=1$, $\alpha=2$, *where N denotes the dimension of x-space.*
This problem was considered in Fujita H. [1] and in more general form [2]. The conclusions of [1] are as follows.

In case of $N\alpha<2$ there does not exist a global solution for any nontrivial nonnegative initial data. On the other hand, in case of $N\alpha>2$, there exists a global solution for sufficiently small initial data, and no global solutions for sufficiently large initial data.

This paper will give a partial settlement for the case $N\alpha=2$.

We consider the next problem.

$$\frac{\partial}{\partial t}u(t,x)=\Delta u(t,x)+u(t,x)^{1+\alpha} \qquad (t,x)\in[0,T)\times R^N, \tag{1}$$
$$u(0,x)=u_0(x),$$

where $u_0(x)$ is a nonnegative bounded continuous function.
A function $u=u(t,x)$ is said to be a solution of (1) if the following (i) and (ii) hold (see [1] or [2]);

(i) $u$ is bounded and continuous in $[0,T']\times R^N$, where $T'$ is an arbitrary constant $<T$. The initial condition is satisfied in the usual sense.

(ii) The differential equation is satisfied by $u$ in the distribution sense in $(0,T)\times R^N$.
The "global solution" means the solution of (1) for $T=\infty$.

**Theorem.** *In case of* $N=2$, $\alpha=1$ *or* $N=1$, $\alpha=2$, *the initial value problem* (1) *has no global solutions for any nontrivial initial data* $u_0$.

The remainder of this paper will be devoted to the proof.

The problem (1) is equivalent to the following problem of the integral equation.

$$\begin{aligned} u(t,x)=&(4\pi t)^{-N/2}\int_{R^N}\exp\left(-|x-y|^2/4t\right)u_0(y)dy \\ &+\int_0^t(4\pi(t-\tau))^{-N/2}\int_{R^N}\exp\left(-|x-y|^2/4(t-\tau)\right) \\ &\times(u(\tau,y))^{1+\alpha}dyd\tau. \end{aligned} \tag{2}$$