107. *Limits of the Discrete Series for the Lorentz Groups*

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(Comm. by Kôsaku YOSIDA, M. J. A., July 12, 1973)

1. Introduction. The purpose of this paper is to construct limits of the discrete series for the Lorentz group of n-th order and to show that the limits are imbedded in the principal series.

Limits of the discrete series have been constructed by Bargmann [1] for $SL(2, \boldsymbol{R})$ and by Takahashi [5] for the De Sitter group. The results in this paper is a generalization of them. Knapp and Okamoto [3] have discussed the same problem for limits of the holomorphic discrete series for a simple Lie group whose associated symmetric space has an invariant complex structure.

The author wishes to thank Professors O. Takenouchi and K. Kumahara for their helpful suggestions.

2. Preliminaries. We denote by $Spin(n, 1)$ the universal covering group of the Lorentz group $SO_e(n, 1)$. $Spin(n, 1)$ has been realized as a group consisting of 2×2 matrices with coefficients in the Clifford algebra by Takahashi [6] as follows: We use the same definitions and notations as in [6]. Let G be the set of matrices $g=\begin{pmatrix} a & b \\ b' & a' \end{pmatrix}$ such that

(2.1) $$a, b \in T_{n-1}, b\bar{a}' \in V_{n-1} \quad \text{and} \quad |a|^2-|b|^2=1.$$

Then G is a group, and if $n\geq 3$ G is isomorphic with $Spin(n, 1)$. If $n=2$, G is isomorphic with $SU(1, 1)$.

The subgroup K of G consisting of matrices $\begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}$ with $k \in T^0_{n-1}$ is isomorphic with $Spin(n)$ and is a maximal compact subgroup of G. We identify $k \in T^0_{n-1}$ with $\begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \in K$ in the sequel.

3. Principal series. Let $G=KAN$ be the Iwasawa decomposition of G, and M the centralizer of A in K. Then the subgroups A, N and M consist of matrices of the form

$$a_t=\begin{pmatrix} \operatorname{ch} t/2 & \operatorname{sh} t/2 \\ \operatorname{sh} t/2 & \operatorname{ch} t/2 \end{pmatrix} (t \in R), \begin{pmatrix} 1-z & z \\ -z & 1+z \end{pmatrix} (z \in E_{n-1})$$

and

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} (m=m' \in T^0_{n-1}),$$

respectively. M is isomorphic with $Spin(n-1)$. Let U and X be the spaces of $x \in V_{n-1}$ such that $|x|=1$ and $|x|<1$, respectively, then G acts