

105. On Prehomogeneous Compact Kähler Manifolds

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1. In this note we establish some results on classification of compact complex prehomogeneous Kähler manifolds. Details will appear elsewhere. By a compact prehomogeneous manifold, we mean a compact complex manifold whose automorphism group has an open orbit. In [4], J. Potters classified prehomogeneous compact complex surfaces. In what follows we shall state a couple of structure theorems on prehomogeneous compact Kähler manifolds and a classification of such manifolds with coirregularity less than 3.

For convenience sake, we list here some notations and terminologies used below. Let V be a compact complex manifold.

$\text{Aut}^\circ(V)$ = the connected biholomorphic automorphism group of V .

$A(V)$ = the Albanese torus of V .

$q(V)$ = $\dim H^1(V, \mathcal{O})$ which is called the irregularity of V .

$cq(V)$ = $\dim V - q(V)$ which is called the coirregularity of V .

By a regular manifold we mean a compact complex manifold whose irregularity vanishes. For a complex analytic vector bundle E on V , we denote by $P(E)$ the projective bundle associated with E .

2. First we state certain general theorems on prehomogeneous manifolds. The following Proposition 1 can be proved by using a lemma due to R. Remmert and van de Ven (see, Potters [4]).

Proposition 1. *A compact complex prehomogeneous manifold is a locally trivial analytic fibre bundle over a compact complex solvmanifold whose fibre is prehomogeneous with trivial Albanese torus.*

Corollary. *A compact Kähler prehomogeneous manifold V is a holomorphic fibre bundle over its Albanese torus $A(V)$ with a regular prehomogeneous fibre.*

In fact every Kähler solvmanifold is isomorphic to a complex torus.

In what follows we always assume that V is Kähler.

Proposition 2. *If $q(V)=0$, then V is a unirational projective variety.*

Proof. For the projectivity of V , see Oeljekraus [3]. We prove the unirationality. Since V is regular, V can be imbedded into a projective space P^n such that this imbedding induces an inclusion of $G = \text{Aut}^\circ(V)$ into $PGL(n)$. This shows that G and its stabilizer subgroup at every point of V are both linear algebraic groups. Since by as-