132. *The Nonlinear Abstract Cauchy-Kowalewski Theorem described in the Form of Ranked Spaces*

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Introduction. Many interpretation of Cauchy-Kowalewski theorem can be seen in the various works. Main ones of them (cf. [1] p. 561) are classified as follows; (1) the classical interpretations (cf. [2] p. 16), (2) the generalized interpretation by T. Yamanaka [3] p. 7 or by L.V. Ovsjannikov [4] p. 819 (an immediate extension of Gelfand-Silov's result [5] p. 124), (3) the one by F. Treves [6] p. 77, and (4) the one by L. Nirenberg [1] p. 561. (1) is the one by using majorant series. (2), (3) and (4) are the one for an evolution equation by using Banach spaces scale regarded as a generalized majorant series. We denote it B.S. scale for short. (2) and (4) are the one for the equation with non-analytic coefficients in t. Nonlinear equations are treated only in (3) and (4). Now, let us show the unified interpretation of (1)$\sim$(4) (i.e. a generalization of the method of majorant series) by using ranked space [7] p. 3. Because ranked space (i.e. a generalization of uniform space by using transcendental ranks) is a generalization of B.S. scale in [4] p. 819, which is suitable for the description of conditional convergence (cf. E.R. integral in [7] p. 25) and for the description of the convergence in the set of germs. The elimination of parameter (by the norm) appearing in B.S. scale is aimed (in § 1) in the construction of ranked spaces by which we generalize the Cauchy-Kowalewski theorem to the one including (1), (2), (3) and (4). In § 2 we briefly discuss the relation pertaining Ovsjannikov's Theorem between our ranked space and B.S. scale.

§ 1. Cauchy-Kowalewski solution. **1°.** Let $\vec{x}\equiv(x_1,\cdots,x_n)$. Let $B_\delta^{n+1}=\{(s,x_1,x_2,\cdots,x_n);|s|<\delta,|x_i|<+\infty,i=1,2,\cdots,n\}$, let $\mathcal{F}_\delta^{(c)}$ be a set of continuous functions $C(B_\delta^{n+1})$ and let $\mathcal{F}_\delta\subset\mathcal{F}_\delta^{(c)}$. If the choice of $\mathcal{F}_\delta$ holds a sort of unicity, the equivalent relation $f_1\equiv f_2$ in $\bigcup_{\delta>0}\mathcal{F}_\delta$ for $f_1\in\mathcal{F}_{\delta_1}$ and $f_2\in\mathcal{F}_{\delta_2}$ defined by $f_1=f_2$ in $B^{n+1}_{\min(\delta_1,\delta_2)}$ satisfies the three axioms of equivalence. The set $\mathcal{F}_\delta^A$ of the analytic functions in s on B_δ^{n+1} is an example of this $\mathcal{F}_\delta$. The set consisting of the equivalent class $[f]$ for $f\in\bigcup_{\delta>0}\mathcal{F}_\delta$ is denoted by $\mathcal{F}$. The element of $\mathcal{F}$ becomes a germ (in a sense). **2°.** Suppose that $(\alpha f_1)(s,\vec{x})\equiv\alpha f_1(s,\vec{x})\in\mathcal{F}_{\delta_1}$ (for any real number α) and $(f_1+f_2)(s,\vec{x})\equiv f_1(s,\vec{x})+f_2(s,\vec{x})\in\mathcal{F}_{\min(\delta_1,\delta_2)}$ hold for $f_1\in\mathcal{F}_{\delta_1}$ and $f_2\in\mathcal{F}_{\delta_2}$, where $\delta_1,\delta_2>0$. Let $[f_1],[f_2]\in\mathcal{F}$ and let $[f]$ be the