129. A Note on Nonsaddle Attractors

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1. Introduction. We consider a dynamical system whose phase space X is a locally compact and connected metric space. Let M be a compact invariant set of this dynamical system. The purpose of this note is to prove the following:

Theorem. *If M is a nonsaddle positive attractor and $X-M$ contains at least one minimal set, then M is positively asymptotically stable whenever $A^+(M)-M$ is connected where $A^+(M)$ denotes the region of attraction of M.*

Definition of the terminology such as nonsaddle set, attractor, etc. will be given below. First we introduce the following notation.

For an arbitrary point x of X, we denote by:

(1) $C^+(x)$, the *positive half orbit* from x,
(2) $C^-(x)$, the *negative half orbit* from x,
(3) $L^+(x)$, the *positive limit set* of x,
(4) $L^-(x)$, the *negative limit set* of x,
(5) $D^+(x)$, the *positive prolongation* of x,
(6) $D^-(x)$, the *negative prolongation* of x,
(7) $J^+(x)$, the *positive prolongational limit set* of x,
(8) $J^-(x)$, the *negative prolongational limit set* of x.

Definition 1. The set
$$A^+(M)=[x\,;\,x\in X,\ M\supset L^+(x)\neq\emptyset],$$
is called the *region of positive attraction* of M, and the set
$$A^-(M)=[x\,;\,x\in X, M\supset L^-(x)\neq\emptyset]$$
is called the *region of negative attraction* of M.

Definition 2. The set
$$a^+(M)=[x\,;\,x\in X, M\cap L^+(x)\neq\emptyset]$$
is called the *region of positive weak attraction* of M, and the set
$$a^-(M)=[x\,;\,x\in X, M\cap L^-(x)\neq\emptyset]$$
is called the *region of negative weak attraction* of M.

Definition 3. M is called a *positive* (*negative*) *attractor* if $A^+(M)$ ($A^-(M)$) is a neighbourhood of M.

Definition 4. M is called a *saddle set* if there exists a neighbourhood U of M such that every neighbourhood of M contains a point x with the property that: