128. *Some Relative Notions in the Theory of Hermitian Forms*

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In the 'classical' surgery theory on compact manifolds, all Hermitian forms to be considered are nonsingular [5]. However, in recent developments in surgery theory [2], [4], we have encountered a some-what curious situation, in which a homomorphism of rings $h: R \to S$ is given, and Hermitian forms to be considered are *defined over* R and *nonsingular over* S. For example, consider a homomorphism $h: \boldsymbol{Z}[t, t^{-1}] \to \boldsymbol{Z}$ defined by $h(t)=1$. Then it is proven that the 'Witt groups' of $\pm t$-Hermitian forms over $\boldsymbol{Z}[t, t^{-1}]$ which become nonsingular over $\boldsymbol{Z}$ are *isomorphic* to the higher dimensional knot cobordism groups. See [3], [4].

In this note we shall formulate (§2) some basic notions concerning the Hermitian forms of the above type, in the framework of, or as a variant of, Wall's L-theory [5] [6], and discuss some elementary properties. We also give an algebraic proof of a cancellation theorem**) which was proven in [4] by a topological method.

Conventions. We always consider rings with 1, not necessarily commutative, satisfying the condition: *The rank of a free module over the ring is well-defined.* All modules will be finitely generated right modules. Let R be a ring, V a quotient group of $K_1(R)=GL(R)/E(R)$. A basis of a free R-module is *V-equivalent* to another basis if the transformation matrix is *V-simple*, in other words, if it represents the zero element of V. A free module with a fixed V-equivalence class of bases is said to be *V-based*, and any basis in the class is called a *V-preferred basis*. We sometimes omit the prefix 'V-' if it is obvious in the context.

1. u-quadratic forms (The main reference is [5].). We fix a ring R with (additive) involution $a \mapsto \bar{a}$ such that $\overline{ab}=\bar{b}\bar{a}$, and $\bar{\bar{a}}=a$ ($\forall a, b \in R$). Note that $\bar{1}=1$. A unit u is *admissible* if $u \in \text{Center}(R)$ and $\bar{u}=u^{-1}$. Let M be an R-module, u an admissible unit. A *u-quadratic form* (λ, μ) *on* M consists of functions $\lambda: M \times M \to R$, $\mu: M \to R/\{a-\bar{a}u|$

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**) Cappell-Shaneson has also given a proof [2, Lemma 1.3]. However, a property of S-isometries (in our terminology) in their proof does not seem to be so trivial as they asserted. It will be proven in the present paper, Theorem 3.