126. *Complex Structures on* $S^1 \times S^5$

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1. Let X be a compact complex manifold of dimension 3 of which the 1st Betti number is equal to 1 and the 2nd Betti number vanishes. X has at most two algebraically independent meromorphic functions. In this note we restrict ourselves to the case where there are exactly two algebraically independent meromorphic functions. Then X has an algebraic net of elliptic curves. Furthermore we assume that this net has no base points, in other words, there exists a surjective holomorphic mapping f onto a projective algebraic (non-singular) surface V whose general fibre is an (connected, non-singular) elliptic curve. Finally we assume that f is equi-dimensional (see Remark 1). This note is a preliminary report on some results on complex structures of X. Details will be published elsewhere.

2. Proposition 1. *Every fibre of f is a non-singular elliptic curve.*

Proposition 2. *V is either a projective plane or a surface of general type.*

Theorem 1. *There exists an unramified covering manifold W of X such that $W \cup \{one\ point\}$ is holomorphically isomorphic to a 3-dimensional affine variety with an algebraic $\boldsymbol{C}^*$ action.*

Denote by α the linear transformation of N-dimensional complex affine space $\boldsymbol{C}^N$ defined by

$$\alpha : (z_1, \cdots, z_N) \mapsto (\alpha_1 z_1, \cdots, \alpha_N z_N),$$

where $\alpha_1^{a_1} = \cdots = \alpha_N^{a_N} = \beta$ for suitable positive integers a_j ($j=1, \cdots, N$) and $0 < |\beta| < 1$. Then the infinite cyclic group $\langle \alpha \rangle$ generated by α acts on $\boldsymbol{C}^N - \{0\}$ freely and the quotient space $\boldsymbol{C}^N - \{0\}/\langle \alpha \rangle$ is a compact complex manifold.

Using some results of C. Chevally and M. Rosenlicht (see [8]), we obtain

Corollary. *There exists a finite unramified covering manifold of X which is holomorphically isomorphic to a submanifold of $\boldsymbol{C}^N - \{0\}/\langle \alpha \rangle$ for some N and α.*

Let X_t be a small deformation of X. Then we have a small deformation W_t of W corresponding to X_t. By a theorem of H. Rossi [10], we obtain

Theorem 2. *$W_t \cup \{one\ point\}$ has a complex structure and be-*