157. *Vanishing Theorems with Algebraic Growth and Algebraic Division Properties*

Complex Analytic De Rham Cohomology. I

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The purpose of the present note is to announce certain quantitative properties of coherent sheaves and analytic varieties. Details will appear elsewhere. Results given here are originally and primarily intended for applications to differential forms on complex analytic varieties with arbitrary singularities (cf. the end of this note). Results stated here are, however, of their own interests. Our basic purpose is to discuss vanishing theorems of certain types where quantitative properties of objects considered appear. Quantitative properties examined here are as follows: (I) Asymptotic behaviors with respect to pole loci. (II) Division properties with respect to subvarieties. Our arguments will be divided into two steps: (i) The step in which only the asymptotic behavior enters. (ii) The step where both asymptotic behaviors and division properties appear.

Notational remarks. We write linear functions and monomials as L and M. A couple, denoted by $\sigma=(\sigma_1,\sigma_2)$, is a couple of positive numbers. For a set $\{\sigma^1,\cdots,\sigma^s\}$ of couples maps $\mathcal{L}:\{\sigma^1,\cdots,\sigma^s\}\to\sigma'$ and $\mathcal{M}:\{\sigma^1,\cdots,\sigma^s\}\to\varepsilon\in \boldsymbol{R}$ are said to be of *exponential-algebraic type* ((*e.a*)-*type*) if $\sigma'=\{M_1(\sigma_1^1,\cdots,\sigma_1^s)\times\exp M_2(\sigma_2^1,\cdots,\sigma_2^s), L(\sigma_2^1+\cdots+\sigma_2^s)\}$, $\varepsilon=M_1(\sigma_1^1,\cdots,\sigma_1^s)\times\exp M_2(\sigma_2^1,\cdots,\sigma_2^s)$.

(I) We start with a datum $(\varDelta(r;P_0),X,D)$ of a polydisc $\varDelta$ with the center P_0 of radius r in $\boldsymbol{C}^n$, a variety* $V\ni P_0$ in $\varDelta$ and a divisor $D\ni P_0$ in $\varDelta$. We write irreducible decompositions of X and D at P_0 as $X_{P_0}=\bigcup_j X_{P_0j}$ and $D_{P_0}=\bigcup_j D_{P_0j}$. Assume that D contains the singular locus of X and that $X_{P_0j}\neq D_{P_0j'}$ for any pair (j,j'). Moreover, consider a coherent sheaf $\mathfrak{F}$ admitting a resolution of the following form

$$0\longrightarrow\mathcal{O}^{d_s}\longrightarrow\cdots\xrightarrow{K_2}\mathcal{O}^{d_1}\xrightarrow{K_1}\mathfrak{F}(\subset\mathcal{O}^d)\longrightarrow 0,$$

where K's are matrices whose coefficients are meromorphic functions on X with the pole $D'=D\cap X$. A point P is near P_0 if P is in a small neighborhood of P_0. For a point near P_0, the intersection $\varDelta(r;P)\cap X$ is denoted by $\varDelta(r;P,X)$. Moreover, for a point $Q\in\varDelta(r;P,X)-D$, we

*) A variety and a function are always complex analytic ones in this note.