152. *Some Radii of a Solid Associated with Polyharmonic Equations*

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(Comm. by Kinjirô KUNUGI, M. J. A., Nov. 12, 1973)

Introduction. In the preceding paper [1], we treated some quantities of a bounded domain in R^2 which we called polyharmonic inner radii. In the present paper, we deal with the similar quantities of a bounded domain in R^3 which is bounded by finite number of regular surfaces. G. Pólya and G. Szegö [2] defined the inner radius of a bounded domain using the Green's function of the domain relative to the Laplace's equation $\Delta u=0$ and they calculated the inner radius of a nearly spherical domain. The aim of this paper is to extend the above results. In the first place, we obtain the Green's function of a sphere relative to the n-harmonic equation $\Delta^n u=0$ and define the n-harmonic inner radius of a bounded domain. In the next place, we compute the n-harmonic inner radius of a nearly spherical domain and it is noticeable that it is monotonously decreasing with respect to integer n.

1. Inner radii associated with polyharmonic equations.

We use the following notations in this section. Let V be a bounded domain in R^3, S the surface of V, P_0 an inner point of V, P the variable point in V and r the distance from P_0 to P.

Definition 1. If a function $u(P)$ satisfies the following two conditions, $u(P)$ is called the Green's function of V with the pole P_0 relative to the n-harmonic equation $\Delta^n u=0$.

(1) In a neighborhood of P_0, $u(P)$ has the form

$$u(P)=r^{2n-3}+h_n(P),$$

where $h_n(P)$ satisfies the equation $\Delta^n h_n=0$ in V and all its derivatives of order $\leqq 2n-1$ are continuous in $V+S$.

(2) All the normal derivatives of order $\leqq n-1$ of $u(P)$ vanish on S.

We can find the Green's function relative to the equation $\Delta^n u=0$ for a sphere in the explicit form.

Theorem 1. *Let V be the sphere of radius R with the center O. If $P_0\neq O$, denoting ρ the distance from O to P_0, P_0' the inversion of P_0 with respect to S and r' the distance from P_0' to P, the Green's function $G_n(P,P_0)$ of V with the pole P_0 relative to the equation $\Delta^n u=0$ is as*