151. A Note on the Perturbing Uniform Asymptotically Stable Systems

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1. Introduction. Recently, A. Strauss and J. A. Yorke [2], [3] obtained results concerning the perturbation of the eventual uniform asymptotic stability (abbreviated by *EvUAS*). They considered

(E) $x'=f(t,x)$

(P–1) $y'=f(t,y)+g(t,y)$

(P–2) $y'=f(t,y)+h(t)$

(P–3) $y'=f(t,y)+g(t,y)+h(t)$

under the condition that $f(t,x)$ satisfies the Lipschitz condition and $g(t,x)$ and $h(t)$ satisfy various conditions. In this note we consider these problems under the generalized Lipschitz conditions.

We shall assume that x, f, g and h are n-vectors in R^n and $|\cdot|$ is some n-dimensional norm. Moreover we shall assume that $x=0$ is *EvUAS* for (E) and g and h are smooth for the local existence.

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2. Definitions and auxiliary lemma. In what follows, we denote by $x(t;t_0,x_0)$ any solution of (E) through the point (t_0,x_0).

Definition 2.1. The origin 0 of R^n is said to be for the system (E):

(E_1) *eventually uniformly stable* (*EvUS*) if, for every $\varepsilon>0$, there exist $\alpha=\alpha(\varepsilon)\geqq 0$ and $\delta=\delta(\varepsilon)>0$ such that

$$|x(t;t_0,x_0)|<\varepsilon \qquad \text{for } |x_0|<\delta \text{ and } t\geqq t_0\geqq\alpha;$$

(E_2) *eventually uniformly attracting* (*EvUA*) if, there exist $\delta_0>0$ and $\alpha_0\geqq 0$ and if for every $\varepsilon>0$ there exists $T=T(\varepsilon)\geqq 0$ such that

$$|x(t;t_0,x_0)|<\varepsilon \qquad \text{for } |x_0|<\delta_0,\ t_0\geqq\alpha_0 \text{ and } t\geqq t_0+T;$$

(E_3) *eventually uniform-asymptotically stable* (*EvUAS*) if (E_1) and (E_2) hold simultaneously.

Definition 2.2. A continuous function $h:[0,\infty)\to R^n$ is said to be *absolutely diminishing* if

$$\int_t^{t+1}|h(s)|ds\to 0 \qquad \text{as } t\to\infty.$$

A continuous function $g:[0,\infty)\times R^n\to R^n$ is said to be *absolutely diminishing* if for some $r>0$ and every $m(0<m<r)$, there exists an