149. *On a Theorem of F. DeMeyer*

By Takasi NAGAHARA
Department of Mathematics, Okayama University

(Comm. by Kenjiro SHODA, M. J. A., Nov. 12, 1973)

Throughout this paper, all rings will be assumed commutative with identity element, and given any ring S, $B(S)$ will mean the Boolean algebra consisting of all idempotents of S. Moreover, R will mean a ring, and all ring extensions of R will be assumed with identity element 1, the identity element of R. Further, $R[X]$ will mean the ring of polynomials in an indeterminate X with coefficients in R, and all monic polynomials will be assumed to be of degree $\geqq 1$. Given a monic polynomial f in $R[X]$, a ring extension S of R is called a splitting ring of f (over R) if $S=R[\alpha_1, \cdots, \alpha_n]$ and $f=(X-\alpha_1)\cdots(X-\alpha_n)$ (cf. [4, Definition]). A polynomial $f \in R[X]$ is called separable if f is monic and $R[X]/(f)$ is a separable R-algebra. In [3], F. DeMeyer introduced the notion of uniform separable polynomials. By [5, Theorem 3.3], it is seen that a separable polynomial $f \in B[X]$ is uniform if and only if f has a splitting ring S which is projective over R and with $B(S)=B(R)$.

In [3], F. DeMeyer stated the following theorem:

Let R be a regular ring (in the sense of Von Neumann) and let S be a finite projective separable extension of R with $B(S)=B(R)$. Then there is an element $\alpha \in S$ and a separable polynomial $p(X) \in R[X]$ so that $S=R[\alpha]$ and α is a root of $p(X)$. Moreover, if S is a weakly Galois extension of R then the polynomial $p(X)$ can be chosen to be uniform ([3, Theorem 2.7]).

However, the proof contains an error which is the statement "Applying the usual compactness argument and decomposing R by a finite number of orthogonal idempotents e as above gives the first assertion of the theorem". Indeed, applying the usual compactness argument, we obtain a polynomial $p(X)$ of $R[X]$ so that $R[X]/(p(X))$ (R-separable)$\sim S$; but if S has not $\mathrm{rank}_R S$ (in the sense of [1, Definition 2.5.2]) then $p(X)$ is not monic, and so, is not separable over R.

The purpose of this note is to improve on the result of the above theorem. First, we shall prove the following lemma which is useful in our study.

Lemma. *Let K be a field, L a field extension of K which is finite dimensional separable, and $L=K[\alpha]$. Let $n \geqq \mathrm{rank}_K L$ be an integer. Then, there exists a monic polynomial $g(X)$ in $K[X]$ of degree n so that $g(\alpha)=0$ and $g(X)$ has no multiple roots (whence $g(X)$ is separable over*