147. *Wave Equation with Wentzell's Boundary Condition and a Related Semigroup on the Boundary. I*

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(Comm. by Kôsaku YOSIDA, M. J. A., Nov. 12, 1973)

1. For *diffusion equation* in one dimension, W. Feller [1] determined all types of possible boundary conditions. A part of the result was extended by A. D. Wantzell [2] for multi-dimensional case. In fact, he found a *candidate* for the most general boundary condition which is possible for diffusion equation

$$\frac{\partial u}{\partial t}=Au, \tag{1}$$

where A is a second order elliptic differential operator on a compact domain $\bar{D}$ in R^N. More precisely, he proved that, under regularity conditions on A and D, any smooth function u in the domain of the generator, which is a contraction of $\bar{A}$, of a strongly continuous semigroup $\{T_t, t\geqslant 0\}$ of non-negative linear operators on $C(\bar{D})$ with norm $\|T_t\|\leqslant 1$,[1] necessarily satisfies a boundary condition:

$$Lu(x)=0, \qquad x\in\partial D.$$

$$\begin{aligned} Lu(x)=&\sum_{i,j=1}^{N-1}\alpha_{ij}(x)\frac{\partial^2 u}{\partial\xi_x^i\partial\xi_x^j}(x)+\sum_{i=1}^{N-1}\beta_i(x)\frac{\partial u}{\partial\xi_x^i}(x)\\ &+\gamma(x)u(x)+\delta(x)Au(x)+\mu(x)\frac{\partial u}{\partial n}(x)\\ &+\int_{\bar{D}}\left\{u(y)-u(x)-\sum_{i=1}^{N-1}\frac{\partial u}{\partial\xi_x^i}(x)\xi_x^i(y)\right\}\nu(x,dy),\end{aligned} \tag{2}$$

where $\{\alpha_{ij}(x)\}$ is symmetric and non-negative definite, $\gamma(x)$, $\delta(x)$, $-\mu(x)$ are non-positive, and $\nu(x,\cdot)$ is a measure on $\bar{D}$. $\{\xi_x^i(y), 1\leqslant i\leqslant N\}$ is a system of functions in $C^2(\bar{D})$ and is a local coordinate in a neighbourhood of x, and $(\partial u/\partial\xi_x^N)(x)$ coincides with the inner normal derivative $(\partial u/\partial n)(x)$.[2] We sometimes omit the suffix x of ξ_x^i. Wentzell also proved that the boundary condition (2) is actually possible in an important special case. For the problem to solve (1) with his boundary condition in general case, we considered a method in [3], [4], which reduces the problem to solve an integro-differential equation on the

1) Here, the domain of the definition of A is $C^2(\bar{D})$ and $\bar{A}$ is the closure of A in $C(\bar{D})$.

2) For a more detailed information about the terms in $Lu(x)$, the reader can consult [2].