145. On the Singularity of the Spectral Measures of a Semi-Infinite Random System

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1. Introduction. H. Matsuda and K. Ishii [1] showed an exponential growth character of polynomials related to a second order difference operator with random coefficients by invoking a limit theorem of H. Furstenberg [4]. A. Casher and J. L. Lebowitz [3] then used this character to derive the singularity of the related spectral measure. We refer the reader to K. Ishii [2] for an improvement of the proof of [3] and for the related physical problems.

The purpose of this note is to simplify the proof of the Matsuda-Ishii theorem and to give an extension of Ishii's results. Let $(\Omega, \mathscr{B}, P)$ be a probability space on which are defined independent real random variables $\{\nu_n(\omega)\}_{n=0}^{\infty}$ with common distribution ν. We consider the following random system on the semi-infinite lattice $Z^+=\{0,1,2,3,\cdots\}$

$$\text{(a)} \qquad \begin{cases} i\dfrac{du_n(t)}{dt}=u_{n-1}(t)-(2+\nu_n)u_n(t)+u_{n+1}(t), \\ u_{-1}(t)=0,\ n\in Z^+,\ t\in[0,\infty). \end{cases}$$

Putting $u_n(t)=y_n e^{-i\lambda t}$, we are led to the following difference equation

$$\text{(b)} \qquad \lambda y_n = y_{n-1}-(2+\nu_n)y_n+y_{n+1},\ n\in Z^+,\ y_{-1}=0.$$

Let $\{p_n^{\omega}(\lambda)\}_{n=0}^{\infty}$ be the solution of (b) under the conditions $y_0=1$ and $y_{-1}=0$. Denote by l_0 the space of all functions on Z^+ with finite supports. We introduce an infinite Jacobi matrix $H^{\omega}=(h_{ij})$, $i, j\in Z^+$, with $h_{ij}=1$, $|i-j|=1$, $h_{ii}=-(2+\nu_i)$, $i\in Z^+$, and $h_{ij}=0$, $|i-j|>1$. $\{H^{\omega}\}$ are regarded as linear operators with domain l_0. Then H^{ω} is an essentially self-adjoint operator on $l^2(Z^+)$ for each $\omega\in\Omega$ and we denote its smallest closed extension by H^{ω} again [5]. We further introduce the resolvent $G^{\omega}(\lambda)=(\lambda-H^{\omega})^{-1}$. Then we have the following expression of $G^{\omega}_{mm}(\lambda)=(G^{\omega}(\lambda)e_m, e_m)$, $m\in Z^+$, [6].

$$G^{\omega}_{mm}(\lambda)=\{p^{\omega}_{mm}(\lambda)\}^2\sum_{i=m}^{\infty}\frac{1}{p_i^{\omega}(\lambda)p_{i+1}^{\omega}(\lambda)}, \qquad \operatorname{Im}\lambda\neq 0.$$

Now let $E^{\omega}(\lambda)$ be the resolution of the identity of H^{ω}. K. Ishii [2] showed that, for almost every fixed $\omega\in\Omega$, $\rho_n^{\omega}(\lambda)=(E^{\omega}(\lambda)e_n, e_n)$, $n\in Z^+$, are singular with respect to the Lebesgue measure $d\lambda$ under the assumption that the support of ν is finite and is not a single point. We will show that this is still true under the weaker assumptions that $\int_{-\infty}^{\infty}|c|\,d\nu(c)<\infty$ and that the support of ν is not a single point