144. *The Noetherian Properties of the Oblique Derivative Problems*

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§0. Introduction and results. In this note we establish the Noetherian properties of the oblique derivative problem, which is a degenerate elliptic boundary value problem studied in Ju. V. Egolov and V. K. Kondrat'ev [1]. They proved, though only partially, existence theorems by using the solution of the Dirichlet problem and elliptic regularization. Our method is different from theirs and our results are more complete and precise. Namely, we reduce the problem to the pseudo differential equation on the boundary, whose principal symbol is Lopatinskian of the considered boundary value problem. In virtue of this we can apply to our problem the regularizer, constructed as in G. I. Èskin [2].

Formulation of the problem. Let Ω be a bounded domain in R^{n+1} with the smooth boundary Γ and Γ_0 be an $(n-1)$-dimensional submanifold of Γ. We consider a second order elliptic differential operator with C^∞-coefficient;

$$L(y,D)=\sum_{k,j=1}^{n+1} a_{kj}(y)D_kD_j+\sum_{k=1}^{n+1} a_k(y)D_k+a(y),$$

$$D_j=\sqrt{-1}\frac{\partial}{\partial y_j}.$$

We assume the following two conditions;

i) $\sum_{k,j=1}^{n+1} a_{kj}(y)\eta_k\eta_j\neq 0$ if $(y,\eta)\in\bar{\Omega}\times(R^{n+1}\backslash 0)$.

ii) For each point $y\in\Omega$ and each pair of linearly independent vectors $\eta,\eta'\in R^{n+1}$, the polynomial $L^0(y,\eta+\tau\eta')$ in complex τ has only one root with negative imaginary part. Here $L^0(y,D)=\sum_{k,j=1}^{n+1} a_{kj}(y)D_kD_j$.

An operator ν is a non-zero C^∞-vector field given in a neighbourhood of Γ satisfying the following two conditions;

i) The coefficient of differentiation in the normal direction appearing in Γ does not vanish in $\Gamma-\Gamma_0$. However, it vanishes on Γ_0.

ii) The restriction of ν to Γ_0 is not a tangent vector of Γ_0.

We shall consider the following boundary value problem; $Lu=f$ in Ω, $\frac{\partial u}{\partial\nu}=g$ on Γ. From the assumption i), we remark that this boundary