143. *Theorems on the Finite-dimensionality of Cohomology Groups.* IV

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In this note we discuss theorems related to the finite-dimensionality of cohomology groups attached to a differential complex. The main purpose of § 2 is to derive the vanishing of the cohomology groups from their finite-dimensionality by deforming the boundary suitably. Note that the result in § 1 is rather isolated from the main stream of the results in this series of our notes in its nature, since the scope of its applicability is completely restricted to the system of linear differential equations with constant coefficients. We have included it here because of its importance. The details and complete arguments will be given somewhere else.

§ 1. In this section we treat exclusively the system of linear differential equations with constant coefficients. First let us recall the following notion of k-convexity (of an open set in $\boldsymbol{R}^n$) due to Palamodov [5].

Definition (Palamodov [5] Part II § 11). An open set Ω in $\boldsymbol{R}^n$ is said to be completely k-convex if it satisfies the following condition:

There exists $h(x)$ in $C^2(\Omega)$ such that its Hessian matrix Hess $h(x)$ $=\{\partial^2 h/\partial x_i \partial x_j\}_{1\leq i,j\leq n}$ has at least k positive eigenvalues and that K_c $=\{x \in \Omega\,;\, h(x)\leqq c\}$ is compact for any $c \in \boldsymbol{R}$.

Remark. The notion of complete k-convexity of Ω introduced above has nothing to do with that of q-convexity of the system of pseudo-differential equations introduced in Sato-Kawai-Kashiwara [6] Chapter III § 2.3, though both notions have stemmed from the notion of q-convexity employed in the theory of analytic functions of several complex variables. In § 2 we use the notion of q-convexity of the system of pseudo-differential equations.

Theorem 1. *Let $\mathcal{M}$ be a system of linear differential equations of finite order with* ***constant*** *coefficients. Take a completely $(n-k)$-convex open set Ω in $\boldsymbol{R}^n$. Then*

$$\mathrm{Ext}^j_{\mathcal{D}}(\Omega\,;\mathcal{M},\mathcal{B})=0 \qquad \text{for } j>k \tag{1}$$

holds. Here $\mathcal{B}$ and $\mathcal{D}$ denote the sheaf of hyperfunctions and linear differential operators, respectively.

This result is a generalization of the result of Komatsu concerning