177. *On a Decomposition of Automorphisms of von Neumann Algebras*

By Hisashi CHODA
Department of Mathematics, Osaka Kyoiku University

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1. Recently, in [1], Borchers has characterized inner automorphisms of von Neumann algebras. In the paper, he also investigated and classified general automorphisms of von Neumann algebras. As an interesting consequence of Theorem 3.8 and Theorem 4.1 in [1], we have the following theorem (cf. Remark in below).

Theorem. *Let $\mathcal{A}$ be a von Neumann algebra, $\mathcal{Z}$ the center of $\mathcal{A}$, α an automorphism of $\mathcal{A}$ and $\mathcal{B}$ the fixed algebra of α, that is, $\mathcal{B}=\{A \in \mathcal{A}; \alpha(A)=A\}$. Then there exists a sequence of mutually orthogonal projections $\{E_n ; n=0, 1, 2, \cdots\}$ in $\mathcal{Z} \cap \mathcal{B}$ which satisfies the following conditions*:

(1) $\sum_{n=0}^{\infty} E_n=I$,

(2) *for each* $n \neq 0$, α^k *is inner on* $\mathcal{A}_{E_n}$ *for* $k \equiv 0 \pmod n$,

(3) *for each* $n \neq 0$, α^k *is freely acting on* $\mathcal{A}_{E_n}$ *for* $k \not\equiv 0 \pmod n$, *and*

(4) α^k *is freely acting on* $\mathcal{A}_{E_0}$ *for* $k=1, 2, \cdots$.

In this paper, we shall show, without applying Theorem 3.8 and Theorem 4.1 in [1], the Theorem using the Kallman decomposition theorem of automorphisms [4: Theorem 1.11].

2. Let $\mathcal{A}$ be a von Neumann algebra and α an automorphism of $\mathcal{A}$ (by an automorphism of a von Neumann algebra we mean an automorphism for the $*$-algebra structure). α is called *freely acting* on $\mathcal{A}$ if

$$AB=\alpha(B)A \qquad \text{for any } B \in \mathcal{A}$$

implies $A=0$ ([4]). If F is a projection in the center of $\mathcal{A}$ fixed by α, we can consider α an automorphism of the reduced von Neumann algebra $\mathcal{A}_F$ of $\mathcal{A}$ by the equality

$$\alpha(AF)=\alpha(A)F \qquad \text{for any } A \in \mathcal{A}.$$

By Kallman's theorem, there exists a central projection F fixed under α such that α is inner on $\mathcal{A}_F$ and α is freely acting on $\mathcal{A}_{I-F}$. We shall call this projection F the *central projection inducing the inner part* of α.

Remark. By Kallman's theorem, we have that α is freely acting on $\mathcal{A}$ if and only if α is outer on $\mathcal{A}_G$ for each central projection G fixed under α. Hence, our Theorem is an immediate result of Theorem