176. *Operator Norms as Bounds for Roots of Algebraic Equations*

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1. Introduction. Very recently, Ifantis and Kouris [1] show, a Hilbert space approach is powerful to give bounds of roots of algebraic equations; actually, they show that the operator bound of a perturbation of the simple unilateral shift by a dyad gives certain bounds of roots. In the present note, giving three norms on n-dimensional vector space, we shall obtain certain bounds of roots estimating operator norms of companion matrices.

For a given algebraic equation

$$p(z)=z^n+a_nz^{n-1}+\cdots+a_1=0, \tag{1}$$

we associate the *companion matrix*

$$T=\begin{pmatrix} -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_2 & -a_1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ & \cdots\cdot & & \cdots & \cdots & \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}, \tag{2}$$

cf. [2], esp. Chapter VII. It is well-known that *the spectrum* $\sigma(T)$ *of* T *coincides with the set of all roots of* (1), i.e.

$$\sigma(T)=\{z\,;\,p(z)=0\}. \tag{3}$$

From (3), we have

$$|z|\leqq r(T)\leqq\|T\|, \tag{4}$$

for any root z of (1), where $r(T)$ is the spectral radius of $T: r(T)=\sup_{z\in\sigma(T)}|z|$ and $\|T\|$ is the operator norm of $T: \|T\|=\sup_{\|f\|=1}\|Tf\|$ considering T as an operator on the n-dimensional Banach space H.

2. Carmichael-Mason's theorem. Here we regard H as the n-dimensional unitary space with orthonormal basis $e_1,\cdots,e_n$. For x, $y\in H$, we put $(x\otimes y)z=(z,y)x$ for $z\in H$. Then we can express the companion matrix T of (1) as

$$T=V-e_1\otimes u, \tag{5}$$

where

$$u=a_n^*e_1+\cdots+a_1^*e_n \tag{6}$$

and