175. *Index Theorem for a Maximally Overdetermined System of Linear Differential Equations*

By Masaki KASHIWARA
Research Institute for Mathematical Sciences, Kyoto University

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In this note we state the index theorem for a maximally overdetermined system of linear partial differential equations. The theorem comprises as a special case the already known index theorem for an ordinary differential equation (Kashiwara [2], Komatsu [4] and Malgrange [5]).

1. Local characteristic. Let (S, x) be a germ of an irreducible analytic space. We define the local characteristic $c_x(S)$ by the induction on the dimension of S as follows.

We embed (S, x) into an enclidean space $(C^N, 0)$ and choose a Whitney stratification $S=\cup S_\alpha$ of S. The open stratum of S is denoted by S_0. Let d_α be the dimension of S_α and x_α be a point in S_α. We define $c_x(S)$ inductively by the following formula

$$c_x(S)=\sum_{S_\alpha\neq S_0} c_x(\bar{S}_\alpha)\chi(U_\alpha\cap S_0\cap Z_\alpha)$$

where U_α denotes a sufficiently small open ball with center x_α, Z_α denotes a $(d_\alpha+1)$-codimensional linear variety in a generic position in C^N sufficiently close to x_α, χ denotes the Euler characteristic and the sum extends over all the strata S_α other than S_0.

Proposition. *The definition of a local characteristic $c_x(S)$ is independent of the choice of the embedding $(S, x)\subset(C^N, 0)$ and the stratification.*

We will give the expamples of local characteristics.

Example 1. If (S, x) is non singular, then $c_x(S)=1$.

Example 2. If (S, x) is a hypersurface in C^{n+1} with the isolated singularity at x, then $c_x(S)=1+(-1)^{n-1}\mu$ where μ is the Milnor number of the generic hyperplane section of S through the point x. In particular, for $S=\{x\in C^{n+1}; x_0^{p_0}+\cdots+x_n^{p_n}=0\}$, we have $c_0(S)=1+(-1)^{n-1}(p_1-1)\cdots(p_n-1)$ with $p_0=\max_j p_j$

Example 3. If (S, x) is a curve, then $c_x(S)$ coincides with the multiplicity of S at x.

Example 4. If $S=\{(x, y, z)\in C^3; x^n+y^pz^q=0\}$ (g. c. d. $(p, q, n)=1$ and $p, q, n\geqq 1$), then $c_0(S)=\min(n, p)+\min(n, q)-n$.

2. Index theorem. Let X be a complex manifold, $\mathcal{O}$ be the sheaf of holomorphic functions on X, $\mathcal{D}$ be the sheaf of differential operators