172. *Numerical Experiments on a Conjecture of B. C. Mortimer and K. S. Williams*

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Let p be a rational prime and n a positive integer $\geqq 2$. We denote by $a_n(p)$ the least positive integral value of a for which the polynomial x_n+x+a is irreducible (mod p), and set

$$a_n=\liminf_{p\to\infty} a_n(p).$$

B. C. Mortimer and K. S. Williams [2] have stated the following

Conjecture. *Put $a_2^*=1$ and for $n\geqq 3$ define*

$$a_n^*=\begin{cases}1 & \text{if } n\equiv 0,1 \pmod 3,\\ 2 & \text{if } n\equiv 2 \pmod 6,\\ 3 & \text{if } n\equiv 5 \pmod 6.\end{cases}$$

Then we have $a_n=a_n^*$.

K. S. Williams [5] proved that this conjecture is in fact true for $n=2$ and 3, and Mortimer and Williams [2] verified the conjecture for all $n\leqq 20$ with the aid of a computer. The results of S. Uchiyama [4] show that the conjecture is true whenever n itself is a prime number.

In § 1 of the present paper we shall show that the conjecture is true for all $n\leqq 40$ by making use of an algorithm which is *faster* than the one used in [2]. As to the discriminant D_n of the polynomial $x_n+x+a_n^*$, it is possible to examine the values of it for a fairly wider range of n, and we observe in § 2 some arithmetical properties of D_n that will be of an independent interest. The computations in § 1 were accomplished by the first-named author and those in § 2 were done by the second-named author.

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§ 1. Irreducibility of $x^n+x+a_n^*$ (mod p). Our basic tool is as in [4] the following theorem which is an immediate consequence of the Frobenius density theorem (cf. [1; Chap. IV, § 5]).

Theorem 1. *Let $n\geqq 2$. If there exists some prime p such that $f_n(x)=x^n+x+a_n^*$ is irreducible* (mod p), *then $a_n=a_n^*$.*

Thus, if we can find some prime p such that $f_n(x)$ is irreducible (mod p), then the conjecture of Mortimer and Williams is true for this n. Our algorithm is based on the following three theorems.

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