171. *On the Asymptotic Behaviour of Brauer-Siegel Type of Class Numbers of Positive Definite Quadratic Forms*

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For natural numbers n and D, $H_n(D)$ denotes the class number of positive definite integral matrices of degree n and determinant D, where two matrices A and B are in the same class if and only if $A = {}^tTBT$ holds for some $T \in GL(n, Z)$. $W(n, D)$ denotes $\sum E(S)^{-1}$ with $E(S) = \#\{T \in GL(n, Z) \mid {}^tTST = S\}$, where S runs over representatives of classes of positive definite integral matrices of degree n and determinant D.

In [1] we have proved

Lemma. *For any fixed natural number n, we have*

$$H_n(D) \sim 2W(n, D) \qquad \text{as } D \to \infty.$$

From this lemma we see easily

Theorem 1. *There exists a sequence of natural numbers $\{D(n)\}_{n=1}^{\infty}$ satisfying*

$$H_{n_k}(D_k) \sim 2W(n_k, D_k) \qquad \text{as } \max(n_k, D_k) \to \infty$$

with, for any sequence $(n_k, D_k)_{k=1}^{\infty}$, $D_k > D(n_k)$ for all k.

If moreover n_k is odd and D_k is odd and square-free, then we have

$$(*) \qquad H_{n_k}(D_k) \sim \pi^{-(n_k(n_k+1))/4} \prod_{l=1}^{n_k} \Gamma\left(\frac{l}{2}\right) \prod_{l=1}^{(n_k-1)/2} \zeta(2l) D_k^{(n_k-1)/2}.$$

Our aim in this note is to announce an explicit value of $D(n)$ for odd n;

Theorem 2. *If n_k is odd and $n_k^2/\log\log D_k \to 0$ as $k \to \infty$, then*

$$H_{n_k}(D_k) \sim 2W(n_k, D_k) \qquad \text{as } k \to \infty.$$

If moreover D_k is odd and square-free, then we have $(*)$ *in Theorem* 1.

This theorem is obtained by giving an explicit value of constants c_i and $c_i(\varepsilon)$ except c_{22} in [1]. If c_{22} is explicitly given, then we have an explicit value of $D(n)$ for even n.

Remark 1. There is no essential difficulty to generalize Theorems 1 and 2 to the cases of algebraic number fields.

Remark 2. In our method we can not avoid that $D(n)$ tends to the infinity if $n \to \infty$. But the author does not know whether $\sup_n D(n)$ can be bounded or not. For example, let us consider cases of even unimodular positive definite quadratic forms; then the Siegel formula