169. Theorems on the Finite-dimensionality of Cohomology Groups. V

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The purpose of this note is to present some theorems which assert the stability of cohomolgy groups attached to an elliptic complex $\mathcal{M}$ under the deformation of the domain. The most essential step in our argument is Lemma 1, which seems to be of its own interest.

We use the same notations and terminologies as in our previous notes [2], [3] and do not repeat their definitions. All problems in this note are considered in the real analytic category, that is, the manifold under consideration is real analytic, the (pseudo-) differential operators considered here have real analytic coefficients and so on.

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Throughout this note (except in the last remark) we always assume the following:

(1) $\mathcal{M}$ denotes an elliptic system of linear differential equations defined on M, which admits a free resolution of length d by the sheaf $\mathcal{D}^f$ of linear differential operators of finite order on M.

(2) $\mathcal{M}$ is purely d-dimensional, i.e., $\mathcal{E}xt^j_{\mathcal{D}^f}(\mathcal{M}, \mathcal{D}^f)=0$, $j\neq d$.

In the sequel we denote by $\mathcal{M}'$ the adjoint system of $\mathcal{M}$. The system $\mathcal{M}'$ is by definition the left $\mathcal{D}^f$-Module given by $\mathcal{E}xt^d_{\mathcal{D}^f}(\mathcal{M}, \mathcal{D}^f)\otimes_{\mathcal{O}}(\Omega^n)^{\otimes -1}$ where Ω^n is the sheaf of holomorphic n-forms.

In this note we essentially use the notion of the "negative" tangential system $(\mathcal{N}')_-$ (of pseudo-differential equations) of $\mathcal{M}'$ defined (on the cotangential sphere bundle of the boundary) via the "negative characteristics" of $\mathcal{M}'$. The "negative" tangential system of $\mathcal{M}'$ can be defined in an analogous way to the "positive" tangential system $\mathcal{N}_+$ of $\mathcal{M}$. (Cf. Kashiwara-Kawai [1], Kawai [2].)

Now we have the following lemma, which supplements Lemma 2 of Kawai [3]. (See also Kuranishi [5] for some related topics.)

Lemma 1. *Let $\Omega=\{x\in M\,;\varphi(x)<0\}$ be a relatively compact domain with C^∞-boundary. Assume that the system $\mathcal{N}_+$ is either $(q+1)$-convex or $(q-1)$-concave at any point in its real characteristic variety.*