13. *On a Sequence of Fourier Coefficients*

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§ 1. Let $f(t)$ be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. Let its Fourier series be

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)=\sum_{n=0}^{\infty}A_n(t). \tag{1.1}$$

Then the conjugate series of (1.1) is

$$\sum_{n=1}^{\infty}(b_n\cos nt-a_n\sin nt)=\sum_{n=1}^{\infty}B_n(t). \tag{1.2}$$

Let $\{p_n\}$ be a sequence such that $P_n=\sum_{k=0}^{n}p_k\neq 0$ for $n=0,1,2,\cdots$. A series $\sum_{n=0}^{\infty}a_n$ with its partial sum s_n is said to be summable (N, p_n) to sum s, if

$$\frac{1}{P_n}\sum_{k=0}^{n}p_{n-k}s_k\to s \quad \text{as} \quad n\to\infty.$$

The $(N, p_n)(C, 1)$ method is obtained by superimposing the method (N, p_n) on the Cesàro means of order one.

Throughout this paper, let $\{p_n\}$ *be a sequence such that* $p_n\geqq 0$, $p_n\downarrow$, $P_n\to\infty$, *and we write*

$$\psi(t)=f(x+t)-f(x-t)-l,$$

$$\Psi(t)=\int_0^t|\psi(u)|\,du$$

and $\tau=[1/t]$, *where* $[\lambda]$ *is the integral part of* λ.

§ 2. Varshney [9] proved that if

$$\Psi(t)=o(t/\log t^{-1}) \quad \text{as} \quad t\to+0, \tag{2.1}$$

then the sequence $\{nB_n(x)\}$ is summable $(N, 1/(n+1))(C, 1)$ to l/π. This was generalized by Sharma [6], Singhal [8] and Dikshit [1], respectively, as follows.

Theorem A (Sharma [6]). *If*

$$\Psi(t)=o(t) \quad as \quad t\to+0, \tag{2.2}$$

and, for some fixed $\delta, 0<\delta<1$,

$$\int_t^{\delta}\frac{|\psi(u)|}{u}\log\frac{1}{u}\,du=o(\log t^{-1}) \quad as \quad t\to+0, \tag{2.3}$$

then the sequence $\{nB_n(x)\}$ *is summable* $(N, 1/(n+1))(C, 1)$ *to* l/π.

Remark 1. (2.3) implies (2.2), because