11. Note on Some Whitehead Products

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1. Introduction. For standard generators $\theta \in \pi_q(S^n)$ the problem whether Whitehead products $[\theta, \iota_n]$ are 0 or not has been investigated by various authors [1], [2], [7], [8]. In this note we are concerned with the question whether $[\theta, \iota_n] \in \theta_* \pi_{n+q-1}(S^q)$ or not. Following the Toda notation [13] our main result is stated as follows.

Theorem. *$[\theta, \iota_n]$ does not lie in the image of $\theta_* : \pi_{n+q-1}(S^q) \to \pi_{n+q-1}(S^n)$ for the following θ:*

η_n, $n \equiv 0, 1 \bmod 4$ *and* $n \geqq 5$; η_n^2, $n \equiv 0 \bmod 4$; ν_n, $n \equiv 1, 3 \bmod 8$ *and* $n \geqq 9$ or $n \equiv 0 \bmod 2$ *and* $n \geqq 6$; ν_n^2, $n \equiv 2 \bmod 4$ *and* $n \geqq 6$; σ_n, $n \equiv 1 \bmod 4$ *and* $n \geqq 13$ or $n \equiv 0 \bmod 2$ *and* $n \geqq 10$; $8\sigma_n$, $n \equiv 2 \bmod 4$ *and* $n \geqq 10$; ε_n, $n \equiv 1 \bmod 4$ *and* $n \geqq 13$; $\bar{\nu}_n$, $n \equiv 1 \bmod 4$ *and* $n \geqq 13$; μ_n, $n \equiv 1 \bmod 4$ *and* $n \geqq 13$; ρ_n, $n \equiv 1 \bmod 4$ *and* $n \geqq 21$; κ_n, $n \equiv 1 \bmod 4$ *and* $n \geqq 21$; ω_n, $n \equiv 1 \bmod 4$ *and* $n \geqq 21$; $\bar{\mu}_n$, $n \equiv 1 \bmod 4$ *and* $n \geqq 21$; ζ_n, $n \equiv 0 \bmod 2$ *and* $n \geqq 6$; $\bar{\kappa}_n$, $n \equiv 1 \bmod 4$ *and* $n \geqq 25$ or $n \equiv 0 \bmod 2$ *and* $n \geqq 8$; $\bar{\zeta}_n$, $n \equiv 0 \bmod 2$ *and* $n \geqq 6$; ν_n^*, $n \equiv 0 \bmod 2$ *and* $n \geqq 18$; $\eta_n \sigma_{n+1}$, $n \equiv 0, 1 \bmod 4$ *and* $n \geqq 12$; $\eta_n \mu_{n+1}$, $n \equiv 0 \bmod 4$ *and* $n \geqq 12$; $\eta_n \rho_{n+1}$, $n \equiv 0, 1 \bmod 4$ *and* $n \geqq 20$; $\eta_n \eta^*_{n+1}$, $n \equiv 0 \bmod 4$ *and* $n \geqq 24$; $\eta_n \bar{\mu}_{n+1}$, $n \equiv 0 \bmod 4$ *and* $n \geqq 24$.

Consequently, from a theorem of James [4] we may deduce

Corollary. *There exist no Poincaré complexes of the form $(S^n \cup_\theta e^{q+1}) \cup e^{n+q+1}$, where θ are elements exhibited in Theorem.*

2. Special cases of Toda's propositions. Some of the following lemmas are obtained as corollaries of Propositions 11.10 and 11.11 of Toda [13], but proofs may be given which are based on the results of James [3], Kervaire [6] and Paechter [12].

Lemma 2.1. *For $n \equiv 0 \bmod 4$, $n \geqq 4$, there exists $\tau_{n-1} \in \pi_{2n-1}(S^{n-1})$ such that $E\tau_{n-1} = [\eta_n, \iota_n]$ and $H(\tau_{n-1}) = \eta_{2n-3}^2$.*

Remark. This is obtained from Proposition 11.10, i) of [13] for $\alpha = \eta_{2n-4}$. According to [13], [10] we may take $\tau_3 = \nu' \eta_6$, $\tau_7 = \sigma' \eta_{14}$, $\tau_{11} = \theta'$, $\tau_{15} \equiv \eta^{*\prime} \bmod E\pi_{30}(S^{14})$ and $\tau_{19} = \bar{\beta}$.

Proof. Introduce the diagram