

10. Dimension of the Fixed Point Set of Z_{p^r} -actions

By Katsuo KAWAKUBO
Osaka University

(Comm. by Kenjiro SHODA, M. J. A., Jan. 12, 1974)

§ 1. Introduction. Concerning the dimension of the fixed point set of G -actions, much has been studied [3], [1], [2], [9], [10], [7], and [8]. In this note, we consider a Z_{p^r} -action (M^n, ϕ, Z_{p^r}) on a closed oriented manifold M^n and study the relation between the bordism properties of M^n and the dimension of the fixed point set. If the action is regular, such a problem was studied in [8]. Here we are concerned with general Z_{p^r} -actions.

In order to state the results, we introduce the following notations. Denote by Ω_n the Thom group of all bordism classes $[M^n]$ of closed oriented smooth n -manifold M^n . Let $\Omega(4j)$ be the subring of $\Omega_* \otimes Z_p$ generated by $\{\Omega_0, \Omega_4, \Omega_8, \dots, \Omega_{4j}\}$. Let $F(Z_{p^r}, k)$ be the subring of $\Omega_* \otimes Z_p$ generated by those bordism classes which are represented by a manifold admitting a Z_{p^r} -action such that the dimension of the fixed point set is less than or equal to k .

Then we have

Theorem. (1) $F(Z_{p^r}, 4k) = F(Z_{p^r}, 4k+1) = \Omega(4kp^r + 2p^r - 2)$

(2) $F(Z_{p^r}, 4k+2) = F(Z_{p^r}, 4k+3) = \Omega(4kp^r + 4p^r - 4)$.

Remark. If $k = -1$, then Theorem means the main result of Conner-Floyd [4].

Corollary 1. *Let (M, Z_{p^r}) be a Z_{p^r} -action. If $[M]$ is indecomposable in $\Omega_* \otimes Z_p$, then there exists a component of the fixed point set of dimension greater than or equal to*

$$\frac{\dim M}{p^r} - 2.$$

Corollary 2. *Each element $x \in \Omega_m$ has a representative which admits a Z_{p^r} -action with fixed point set of dimension less than or equal to m/p^r .*

Throughout this paper, p denotes an odd prime integer.

The results in this paper are oriented bordism versions of the excellent papers [5], [7] of tom Dieck.

Detailed proof will appear elsewhere.

§ 2. Outline of the proof. The following diagram is an oriented bordism version of tom Dieck [5],