7. *On a Relation between Characters of Discrete and Non-Unitary Principal Series Representations*

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§ 1. **Introduction.** For the general linear group $G=SL(2,R)$, it was proved by I. M. Gelfand and M. I. Graev, N. Ya Vilenkin in [6] that the quotient representation of certain non-unitary principal series representations by its finite dimentional invariant subrepresentation is infinitesimaly equivalent to a representation which belongs to the discrete series.

Our purpose is to prove a similar relation for any group G satisfying the following conditions:

(C.1) G is a connected real simple Lie group.

(C.2) There is a simply connected complex simple Lie group G_c which is the complexification of G.

(C.3) The symmetric space G/K is of rank one and G has a compact Cartan subgroup, where K denotes the maximal compact subgroup of G.

In § 3, we prove the relation using the explicit character formulas for the representations in discrete series and in non-unitary principal series obtained by Harish-Chandra ([2], [4], [5]).

In § 4, we state some results for $G=\mathrm{Spin}\ (2l,1)$ $(l\geqq 1)$ using Theorem 1.

§ 2. **Preliminaries.** Let G be a Lie group satisfying conditions C.1, C.2 and C. 3 with Lie algebra $\mathfrak{g}$. We shall always denote by $\mathfrak{L}_c$ the complexification of Lie sub-algebra $\mathfrak{L}$ of $\mathfrak{g}$. By C.2, $\mathfrak{g}_c$ is the Lie algebra of G_c.

Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be a Cartan decomposition and K be the analytic subgroup of G whose Lie algebra is $\mathfrak{k}$. We shall fix a Cartan subalgebra $\mathfrak{b}(\subset\mathfrak{k})$ of $\mathfrak{g}$. Let Ω be the non-zero root system of $\mathfrak{g}_c$ with respect to $\mathfrak{b}_c$. For any root α, we can select a root vector X_α such that $B(X_\alpha,X_{-\alpha})=1$ (Where B is the Killing form of $\mathfrak{g}_c$). As usual we identify $\mathfrak{b}_c$ with the dual space of $\mathfrak{b}_c$ by the relation $\lambda(H)=B(H,H_\lambda)$ and denote $(\lambda,\mu)=B(H_\lambda,H_\mu)$ for two linear functions λ,μ on $\mathfrak{b}_c$. Then we have $[X_\alpha,X_{-\alpha}]=H_\alpha$ for any root $\alpha\in\Omega$. For a fixed non-compact root γ, we select a compatible ordering in dual space of RH_γ and $\sqrt{-1}\mathfrak{b}$ such that $\gamma>0$. Put