4. *A Proof of Ehrenpreis' Fundamental Principle in Hyperfunctions*

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1. Introduction. The purpose of this note is to give another easier proof of the theorem of integral representation for hyperfunction solutions of linear partial differential equations with constant coefficients which was first formulated and proved in Kaneko [3], [4].

Most results in the general theory of systems of linear partial differential equations with constant coefficients are deduced from Ehrenpreis' Fundamental Principle (cf. Ehrenpreis [1], [2] and Palamodov [6]), which says the following:

Let $\mathcal{P}$ denote the ring of linear partial differential operators with constant coefficients in n variables. Given an $r_1\times r_0$ matrix $P(D_x)$ with elements in $\mathcal{P}$, we can define a multiplicity variety $\mathfrak{B}$ which is a set of finite pairs of irreducible affine algebraic varieties V_λ in $\boldsymbol{C}^n$ and row vectors $\partial_\lambda(\zeta, D_\zeta)$ of length r_0 whose elements are differential operators in $\boldsymbol{C}^n$ with polynomial coefficients (which are called noetherian operators in Palamodov [6]). Let $\mathcal{F}$ be a certain function space of $\mathcal{P}$-module. Then every kernel u of the map $P(D_x): \mathcal{F}^{r_0}\to\mathcal{F}^{r_1}$ can be expressed in the form

$$u(x)=\sum_\lambda \int_{V_\lambda} {}^t\partial_\lambda(\zeta, D_\zeta)\exp\langle\sqrt{-1}\,x,\zeta\rangle\, d\mu_\lambda(\zeta), \tag{1}$$

where each μ_λ is a measure with support in V_λ which satisfies some growth conditions at infinity determined by $\mathcal{F}$. The integral converges in the topology of $\mathcal{F}$.

When $\mathcal{F}$ is the space of distributions or infinitely differentiable functions on a convex domain in $\boldsymbol{R}^n$ or holomorphic functions on a convex domain in $\boldsymbol{C}^n$, the above statement is proved by Ehrenpreis [2] and Palamodov [6]. In case $\mathcal{F}$ is the space of hyperfunctions $\mathcal{B}(\Omega)$ on a convex domain Ω in $\boldsymbol{R}^n$, the measures in (1) satisfy

$$\int_{V_\lambda}\exp(-\varepsilon|\zeta|+H_K(\zeta))\,|d\mu_\lambda(\zeta)|<\infty, \qquad \text{for } \forall\varepsilon>0,\ \forall K\subset\subset\Omega, \tag{2}$$

where $H_K(\zeta)=\sup_{x\in K}\operatorname{Re}\langle\sqrt{-1}\,x,\zeta\rangle$. The integral is considered in the sense of hyperfunctions. (See Kaneko [3] or the proof below.) We give a proof in this case using the result in the case when $\mathcal{F}$ is the space of holomorphic functions.

2. Proof. Set $U=\{z\in\boldsymbol{C}^n;\operatorname{Re} z\equiv(\operatorname{Re} z_1,\cdots,\operatorname{Re} z_n)\in\Omega\}$ and $U_i=\{z\in U;\operatorname{Im} z_i\neq 0\}$. Since U and U_i are Stein open sets in $\boldsymbol{C}^n$, Leray's