2. *Eigenfunction Expansions for Symmetric Systems of First Order in the Half-Space* R_+^n

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1. Introduction. Eigenfunction expansion theory by distorted plane waves was initiated by T. Ikebe [1] and has been investigated by many authors, for example, Y. Shizuta [9], N. A. Shenk II [8], K. Mochizuki [6], J. R. Schulenberger and C. H. Wilcox [7] and others. T. Ikebe treated the Schrödinger operator $-\Delta+q(x)$ in the whole 3-dimensional Euclidean space $\boldsymbol{R}^3$. Y. Shizuta treated $-\Delta$ in an exterior domain of $\boldsymbol{R}^3$ and N. A. Shenk II generalized the result to the higher dimensional case (see also T. Ikebe [2]). K. Mochizuki treated symmetric systems in an exterior domain of $\boldsymbol{R}^n$ and J. R. Schulenberger and C. H. Wilcox in the whole space $\boldsymbol{R}^n$. An other approach to spectral representations for the operators associated with the wave equation and symmetric hyperbolic systems in an exterior domain of $\boldsymbol{R}^n$ is developed by P. D. Lax and R. S. Phillips [3]. In this note we consider stationary problems for symmetric hyperbolic systems with constant coefficients in the half-space $\boldsymbol{R}_+^n$ and give an expansion theorem by the improper eigenfunctions for such a problem. We note that this problem cannot be regarded as a perturbation of the whole space problem. In fact, our theory is a generalization of the sine and cosine transformations in the L^2 space on the positive half-line which are eigenfunction expansions for $-\dfrac{d^2}{dx^2}$ with Dirichlet or Neumann conditions.

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2. Assumptions. We denote the n-dimensional Euclidean space by $\boldsymbol{R}^n$ and its point by $x=(x_1, \cdots, x_n)$. We also denote a point in $\boldsymbol{R}^{n-1}$ by $x'=(x_1, \cdots, x_{n-1})$ and the set $\{x \in \boldsymbol{R}^n; x_n>0\}$ by $\boldsymbol{R}_+^n$. Let L be a first order symmetric hyperbolic operator with constant coefficients:

$$L=I\frac{\partial}{\partial t}-\sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j}, \tag{1}$$

where I is the identity matrix of order N and the A_j are $N\times N$ constant Hermitian matrices. We consider the mixed initial and boundary value problem in $\boldsymbol{R}_+^n$ for L: