32. *On Certain L^2-well Posed Mixed Problems for Hyperbolic System of First Order*

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1. Introduction and Theorem. Let P be a x_0-strictly hyperbolic $2p\times 2p$-system of differential operators of first order defined over a C^∞-cylinder $R^1\times\Omega\subset R^{n+1}$. Let B be a $p\times 2p$-system of functions defined on the boundary Γ of $R^1\times\Omega$. We consider the following mixed problems under certain conditions:

$$\begin{aligned} &P(x,D)u=f && x\in R^1\times\Omega && (x_0>0),\\ &B(x)u=g && x\in\Gamma && (x_0>0),\\ &u=h && \text{on } x_0=0 && \end{aligned}$$

where $\sqrt{-1}D=\left(\frac{\partial}{\partial x_0},\frac{\partial}{\partial x_1},\cdots,\frac{\partial}{\partial x_n}\right)$.

For the sake of simplicity of descriptions, we may only consider the case where $\Omega=\{x_n>0\}$, by the localization process. Then our assumptions are the following:

(I) α) The coefficients of P and B are real, belong to $C^\infty(R^1\times\bar{\Omega})$ and constant outside some compact set of $R^1\times\bar{\Omega}$.

β) For P, it satisfies the $\sharp$ condition with respect to Γ and for fixed real (x,τ,σ) there is at most one real double root λ of $|P|(x,\tau,\sigma,\lambda)=0$ where $x\in\Gamma$. Furthermore it is non-characteristic with respect to Γ and it is normal, i.e.

$$|P|(x,0,\sigma,\lambda)\neq 0$$

for any real $(\sigma,\lambda)\neq 0$.

γ) The p row-vectors of $B(x)$ are linearly independent, where $x\in\Gamma$.

(II) α) If the Lopatinsky determinant $R(x_0,\tau_0,\sigma_0)=0$ for a real point (x_0,τ_0,σ_0) such that there are no real double roots λ of $|P|(x_0,\tau_0,\sigma_0,\lambda)=0$, then

$$|R(x_0,\tau_0-i\gamma,\sigma_0)|\geq 0(\gamma^1)\qquad(\gamma>0).$$

Furthermore if there is at least one real simple root $\lambda(x_0,\tau_0,\sigma_0)$, the zero set of $R(x,\tau\pm i\gamma,\sigma)$ in some neighborhood $U(x_0,\tau_0,\sigma_0)$ is in the set $\{\gamma=0\}$.

β) If $R(x_0,\tau_0,\sigma_0)=0$ for a real point (x_0,τ_0,σ_0) such that there are real double roots λ of $|P|(x_0,\tau_0,\sigma_0,\lambda)=0$, then

$$|R(x_0,\tau_0-i\gamma,\sigma_0)|\geq 0(\gamma^{1/2})\qquad(\gamma>0).$$

Furthermore if there is at least one real simple root λ, the rank of the