31. *Characterization of the Well-Posed Mixed Problem for Wave Equation in a Quarter Space*

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§1. **Introduction.** R. Sakamoto [6] and H. O. Kreiss [3] had proved the existence and the uniqueness of a solution for hyperbolic mixed problem in Sobolev space under the uniform Lopatinski's condition. Recently, S. Miyatake [5] obtained the necessary and sufficient condition under which the mixed problem for second order hyperbolic equation with real variable coefficients is L^2-well posed. In the case where the coefficients are constant, R. Sakamoto [7] obtained the necessary and sufficient condition under which we can solve the mixed problem for general higher order hyperbolic equation in C^∞-space.

In this note we try to solve the following hyperbolic mixed problem in $C^\infty(V(t_0))$-space, $V(t_0)=\{(t,x,y);\, t>t_0,\, x>0,\, y\in R^{n-1}\}$,

$$(1.1)\quad \begin{cases} \left(D_t^2-D_x^2-\sum_{i=1}^{n-1}D_{y_i}^2\right)u\equiv\square u=f(t,x,y) & \text{in } V(t_0)\\ (u,D_tu)=(\varphi_0,\varphi_1)=\vec{\varphi}(x,y) & \text{on } V_0(t_0)=\overline{V(t_0)}\cap\{t=t_0\}\\ B(t,y\,;D_t,D_x,D_y)u=g(t,y) & \text{on } V_1(t_0)=\overline{V(t_0)}\cap\{x=0\}, \end{cases}$$

where $B=D_x+b_0(t,y)D_t+\sum_{i=1}^{n-1}b_i(t,y)D_{y_i}+c(t,y)$, and $D_t=-i\dfrac{\partial}{\partial t}$,

$$D_x=-i\frac{\partial}{\partial x},\ D_y=(D_{y_1},\cdots,D_{y_{n-1}})=-i\left(\frac{\partial}{\partial y_1},\cdots,\frac{\partial}{\partial y_{n-1}}\right).$$

We assume that b_0, b_i $(i=1,\cdots,n-1)$ and c belong to $\mathcal{B}^\infty(R^n)$, and that b_0 and b_i $(i=1,\cdots,n-1)$ are real-valued.

If a solution $u(t,x,y)$ of (1.1) belongs to $C^m(\overline{V(t_0)})$, then

$$(1.2)\quad D_t^k(Bu)\Big|_{\substack{t=t_0\\x=0}}=D_t^kg|_{t=t_0},\qquad k=0,1,\cdots,m.$$

If we rewrite (1.2) by using $f,\vec{\varphi}$ and g, we get the compatibility conditions of order m for $f,\vec{\varphi}$ and g.

Definition 1. The mixed problem (1.1) is said to be $\mathcal{E}$-well posed (at $t=t_0$) if the following two properties hold

(E.1) for any $(f,\vec{\varphi},g)\in C^\infty(V(t_0))\times C^\infty(V_0(t_0))^2\times C^\infty(V_1(t_0))$ which satisfy the compatibility conditions of order 2 there exists a unique solution $u(t,x,y)$ of (1.1) in $C^2(V(t_0))$,

(E.2) there exists a positive constant λ such that the value of the solution of (1.1) at $(t_1,x_1,y_1)\in V(t_0)$ depends only on the data in $C_{(t_1,x_1,y_1)}=\{(t,x,y)\in V(t_0);\, t-t_1<-\lambda\,|(x,y)-(x_1,y_1)|\}$.