29. *Remarks on Moduli of Invertible Elements in a Function Algebra*

By Junzo WADA
Waseda University, Tokyo

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Let X be a compact Hausdorff space and let A be a function algebra on X. A theorem of Hoffman-Wermer [4] asserts as follows: the set of real parts $\mathrm{Re}\, A$ of A closed implies $A=C(X)$. On the other hand, E. Gorin [3] proved that if A is a function algebra on a compact metric space and if $\log|A^{-1}|=C_R(X)$, then $A=C(X)$, where A^{-1} denotes the set of invertible elements in A and $|A^{-1}|$ is the moduli of A^{-1}. We here consider the following condition: there is a closed subset N in $C_R(X)$ with $\log|A^{-1}|\supset N\supset \mathrm{Re}\, A$. The hypotheses of two above theorems satisfy the condition. The aim of this paper is to investigate properties of function algebras which have the condition and to give an extension of the Gorin theorem (Corollary 2).

We begin with the following theorem which states relations between the Hoffman-Wermer theorem and the Gorin theorem.

Theorem. *Let X be a compact Hausdorff space and let A be a function algebra on X. Assume that there is a closed linear subspace in $C_R(X)$ with $\log|A^{-1}|\supset N\supset \mathrm{Re}\, A$. If F is a maximal antisymmetric set for A, then the following properties are equivalent.*

(i) *F is metric*

(ii) $N|F=\mathrm{Re}\,(A|F)$

(iii) *F is a single point,*

where $A|F$ and $N|F$ denote the restrictions of A and N the F respectively.

In order to verify the theorem, we shall first show the following lemma which is essentially due to Hoffman-Wermer [4].

Lemma. *Let A be a function algebra on a compact Hausdorff space. Assume that there is a closed linear subspace N in $C_R(X)$ with $N\supset \mathrm{Re}\, A$. If F is a maximal antisymmetric set for A and if $N|F=\mathrm{Re}\,(A|F)$, then F is a single point.*

Proof. If $h\in \mathrm{Re}\,(A|F)$, then from the hypothesis there is an $h^*\in N$ such that $h^*|F=h$. Therefore we have:

$$\varphi\rightarrow\varphi|F$$

is a linear transformation of N onto $\mathrm{Re}\,(A|F)$ whose norm is 1. Let $R=\{f\in N;\, f(F)=0\}$. Then R is a closed linear subspace and the factor space N/R is isometric to $\mathrm{Re}\,(A|F)$. This can be verified by the