27. *Riemannian Manifolds Admitting Some Geodesic*

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1. Introduction. Let M be a compact Riemannian manifold and f an isometry of M. Then a geodesic α on M is called f-invariant geodesic if $f\alpha=\alpha$. It is not known much about isometry invariant geodesic. In this paper we see what kind of Riemannian manifold admits an isometry invariant geodesic. Our results are following;

Theorem A (K. Grove). *Let M be a compact connected, simply connected and oriented Riemannian manifold of odd dimension and f an orientation preserving isometry of M. Then there exists an f-invariant geodesic.*

Theorem B. *Let M be a compact connected, simply connected and oriented Riemannian manifold of $2k$-dimension and f an orientation preserving isometry of M. Then there exists an f-invariant geodesic for $k=1$ and also well for $k>1$ if $\lambda_k(f)=$even where $\lambda_k(f)$ is the trace of an induced homomorphism $f_k: H_k(M, Q)\to H_k(M, Q)$ where Q is the field of rational numbers.*

Corollary. *Let M be a manifold of Theorem B. Then M admits an f-invariant geodesic for any orientation preserving isometry f of M if $H_k(M, Q)=0$.*

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2. Fixed points of isometry. Let M be a compact manifold and f be an isometry of M. Then the induced homomorphism by f of the i-th homology group of M over coefficient Q is denoted by $f_i: H_i(M, Q)\to H_i(M, Q)$ and the trace of f_i by $\lambda_i(f)$.

Lemma 1. *Let M be an n-dimensional orientable Riemannian manifold and f an orientation preserving isometry, then we have $\lambda_i(f)=\lambda_{n-i}(f)$ $(i=1\sim n)$.*

Proof. We have only to use the Poincaré duality. q.e.d.

Lemma 2. *Let M be an odd dimensional orientable Riemannian manifold and f an orientation preserving isometry of M, then f has no isolated fixed points.*

Proof. Let x be a fixed point of f and $f_*: T_x(M)\to T_x(M)$ be an induced homomorphism by f. Then f_* is an element of $SO(n)$ and so f_* has a following representation with respect to a suitable basis;