23. *Oscillation Theorems for a Damped Nonlinear Differential Equation*

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In this paper we are concerned with the oscillatory behavior of solutions of the nonlinear differential equation

(A) $$x^{(n)}(t)+q(t)\phi(x^{(n-1)}(t))+p(t)f(x(g(t)))=0.$$

Our main purpose is to extend to equation (A) some of the recent results regarding oscillation of solutions of the differential equation with a time lag

(B) $$x^{(n)}(t)+p(t)f(x(g(t)))=0$$

and the differential equation without a time lag

(C) $$x^{(n)}(t)+q(t)\phi(x^{(n-1)}(t))+p(t)f(x(t))=0.$$

We consider only solutions $x(t)$ of (A) which exist on some half-line $[T_x,\infty)$. A solution $x(t)$ of (A) is said to be oscillatory (or to oscillate) if $x(t)$ has a sequence of zeros $\{t_k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} t_k=\infty$; otherwise, a solution is said to be nonoscillatory.

Throughout this paper the following assumptions are assumed to hold:

(a) $f\in C(R)\cap C^1(R-\{0\})$, $R=(-\infty,\infty)$, and
$$xf(x)>0,\quad f'(x)\geqq 0\quad \text{for all } x\in R-\{0\};$$

(b) $\phi\in C(R)$, and there is a constant $M>0$ such that
$$0<y\phi(y)\leqq My^2\qquad \text{for all } y\in R-\{0\};$$

(c) $g\in C^1(R^+)$, $R^+=(0,\infty)$, $g(t)\leqq t$, $g'(t)\geqq 0$ for all $t\in R^+$, and $\lim_{t\to\infty} g(t)=\infty$;

(d) $p\in C(R^+)$, and $p(t)>0$ for all $t\in R^+$;

(e) $q\in C(R^+)$, and there is a nonnegative function $m\in C(R^+)$ such that $q(t)\leqq m(t)$ for all $t\in R^+$ and $\lim_{t\to\infty} Q(t,T)=\infty$ for any fixed $T\in R^+$, where
$$Q(t,T)=\int_T^t \exp\left(-M\int_T^s m(u)du\right)ds.$$

Lemma. *Suppose that assumptions* (a)—(e) *hold. If* $x(t)$ *is a nonoscillatory solution of* (A), *then there is a* T *such that* $x(t)x^{(n-1)}(t)>0$ *for all* $t\in[T,\infty)$.

Proof. We may assume that $x(t)>0$ on $[t_0,\infty)$, since a parallel argument holds when $x(t)<0$ on $[t_0,\infty)$. Since $\lim_{t\to\infty} g(t)=\infty$, there is $t_1\geqq t_0$ such that $x(g(t))>0$ on $[t_1,\infty)$. Suppose that there is $t^*\in[t_1,\infty)$ at which $x^{(n-1)}(t^*)=0$. From (A) we see that