46. A Remark on Almost-Continuous Mappings

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1. Introduction. In 1968, M. K. Singal and A. R. Singal [2] defined almost-continuous mappings as a generalization of continuous mappings. They obtained an extensive list of theorems about such a mapping, among them, the following two results were established:

Theorem A. *Let* $f_\alpha: X_\alpha \to X_\alpha^*$ *be almost-continuous for each* $\alpha \in I$ *and let* $f: \Pi X_\alpha \to \Pi X_\alpha^*$ *be defined by setting* $f((x_\alpha)) = (f_\alpha(x_\alpha))$ *for each point* $(x_\alpha) \in \Pi X_\alpha$. *Then* f *is almost-continuous.*

Theorem B. *Let* $h: X \to \Pi X_\alpha$ *be almost-continuous. For each* $\alpha \in I$, *define* $f_\alpha: X \to X_\alpha$ *by setting* $f_\alpha(x) = (h(x))_\alpha$. *Then* f_α *is almost-continuous for all* $\alpha \in I$.

The purpose of the present note is to show that the converses of the above two theorems are also true. As the present author has a question in the proof of Theorem B, we shall give the another proof.

2. Definitions and notations. Let A be a subset of a topological space X. By $\mathrm{Cl}\, A$ and $\mathrm{Int}\, A$ we shall denote the closure of A and the interior of A in X respectively. Moreover, A is said to be regularly open if $A = \mathrm{Int}\,\mathrm{Cl}\, A$, and regularly closed if $A = \mathrm{Cl}\,\mathrm{Int}\, A$. By a space we shall mean a topological space on which any separation axiom is not assumed. A mapping f of a space X into a space Y is said to be *almost-continuous* (simply *a.c.*) if for each point $x \in X$ and any neighborhood V of $f(x)$ in Y, there exists a neighborhood U of x such that $f(U) \subset \mathrm{Int}\,\mathrm{Cl}\, V$. It is a characterization of *a.c.* mappings that the inverse image of every regularly open (resp. regularly closed) set is open (resp. closed) [2, Theorem 2.2]. A mapping is said to be *almost-open* if the image of every regularly open set is open.

3. Preliminaries. We begin by the following lemma.

Lemma 1. *If a mapping* $f: X \to Y$ *is a.c. and almost-open, then the inverse image* $f^{-1}(V)$ *of each regularly open set* V *of* Y *is a regularly open set of* X.

Proof. Let V be an arbitrary regularly open set of Y. Then, since f is *a.c.*, $f^{-1}(V)$ is open and hence we obtain that $f^{-1}(V) \subset \mathrm{Int}\,\mathrm{Cl}\, f^{-1}(V)$. In order to prove that $f^{-1}(V)$ is regularly open, it is sufficient to show that $f^{-1}(V) \supset \mathrm{Int}\,\mathrm{Cl}\, f^{-1}(V)$. Since f is *a.c.* and $\mathrm{Cl}\, V$ is regularly closed, $f^{-1}(\mathrm{Cl}\, V)$ is closed and hence we have $\mathrm{Int}\,\mathrm{Cl}\, f^{-1}(V) \subset \mathrm{Cl}\, f^{-1}(V) \subset f^{-1}(\mathrm{Cl}\, V)$. Since f is almost-open and $\mathrm{Int}\,\mathrm{Cl}\, f^{-1}(V)$ is