43. *Elliptic Boundary Problems in Non-Compact Manifolds. I*

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1. Introduction. We consider boundary problems for elliptic differential equations. When the manifold (with boundary) is compact and its boundary is smooth, the indexes of elliptic boundary problems are finite (see [1], etc.). When the manifold is not compact or the data are not given on the entire boundary, the situation is different. Such cases will be studied in this and the forthcoming papers (see Theorem 3).

Let $\mathfrak{M}$ be a σ-compact C^∞ manifold (without boundary), and Ω an open subset of $\mathfrak{M}$. Let ω be an open subset of the topological boundary of Ω in $\mathfrak{M}$. Then we denote by Ω^* the pair of Ω and ω. We also use Ω^* to denote the union of Ω and ω. Take an open subset Ω_0 of $\mathfrak{M}$ such that Ω is contained in Ω_0 and the intersection of Ω_0 and the boundary of Ω in $\mathfrak{M}$ is equal to ω.

Let $\mathcal{F}(\Omega_0)$ be a subspace of $\mathcal{D}'(\Omega_0)$ with a locally convex topology (For the notation of our function spaces, see [1].). Let ρ be the restriction mapping of $\mathcal{D}'(\Omega_0)$ to $\mathcal{D}'(\Omega)$. Then we denote by $\mathcal{F}(\Omega^*)$ the space $\rho(\mathcal{F}(\Omega_0))$, that is, $\mathcal{F}(\Omega^*)=\{u \in \mathcal{D}'(\Omega)\,;\, u=\rho(u_0) \text{ for some } u_0 \in \mathcal{F}(\Omega_0)\}$. This space is endowed with the strongest locally convex topology such that ρ is continuous from $\mathcal{F}(\Omega_0)$ onto $\mathcal{F}(\Omega^*)$. Next we denote by $\mathring{\mathcal{F}}(\Omega^*)$ the closed subspace of $\mathcal{F}(\Omega_0)$ defined by $\mathring{\mathcal{F}}(\Omega^*)=\{u \in \mathcal{F}(\Omega_0)\,;\, \operatorname{supp} u \subset \Omega^*\}$.

In this paper we assume that ω is of C^∞ class. Let R denote the trace operator of $C^\infty(\Omega^*)$ onto $C^\infty(\omega)$. Take a C^∞ vector field ν in a neighborhood of ω which is not tangential to ω. By D_ν we denote the differentiation in the direction ν. Write $\gamma_m=(R, R\circ D_\nu, R\circ D_\nu^2, \cdots, R\circ D_\nu^{m-1})$, for a natural number m.

2. Function spaces $C^\infty(\Omega^*)$ and $\mathring{C}^\infty(\Omega^*)$. Proposition 1. *The space $C^\infty(\Omega^*)$ is separable Fréchet Montel and its dual space is isomorphic to $\mathring{\mathcal{E}}'(\Omega^*)$.*

Outline of the proof. Since $C^\infty(\Omega_0)$ is a Fréchet-Schwartz space, $C^\infty(\Omega^*)$ is also Fréchet-Schwartz (see [2]). Then the former part of the proposition follows. Moreover the dual space of $C^\infty(\Omega^*)$ is isomorphic to the polar of $C^\infty(\Omega^*)$ in $\mathcal{E}'(\Omega_0)$. Using a result due to Schwartz [5], p. 93, we can easily obtain the latter part of the proposition.

Proposition 2. *Let $s \in \boldsymbol{R}$ and $\chi \in C^\infty(\Omega_0)$. Take a compact subset K_1 in the interior of $K=\operatorname{supp}\chi$. Set*