# 42. Symmetric Spaces Associated with Siegel Domains

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(Comm. by Kôsaku YOSIDA, M. J. A., March 12, 1974)

**Introduction.** Let $D$ be a Siegel domain of the second kind due to Pyatetski-Shapiro [2]. We then construct a symmetric Siegel domain in $\bar{D}$ which is invariant under a suitable equivalence. At the same time we establish a structure theorem of the Lie algebra of all infinitesimal automorphisms of the domain $D$.

**1.** Let $\mathfrak{g}=\sum_p \mathfrak{g}^p$ ($p \in \boldsymbol{Z}, [\mathfrak{g}^p, \mathfrak{g}^q] \subset \mathfrak{g}^{p+q}$) be a graded Lie algebra over $\boldsymbol{R}$ with $\dim \mathfrak{g} < \infty$. Then the radical $\mathfrak{r}$ of $\mathfrak{g}$ is a graded ideal. Concerning Levi decompositions of $\mathfrak{g}$, we obtain

**Theorem 1.** *There exists a semi-simple graded subalgebra* $\mathfrak{s}$ *of* $\mathfrak{g}$ *such that* $\mathfrak{g}=\mathfrak{s}+\mathfrak{r}$.

**2.** Denote by $R$ (resp. by $W$) a real (resp. complex) vector space of a finite dimension, and by $R_c$ the complexification of $R$. Let $D$ be a Siegel domain of the second kind in $R_c \times W$ associated with a convex cone $V$ in $R$ and a $V$-hermitian form $F$ on $W$. We denote by $\mathfrak{g}(D)$ the Lie algebra of all infinitesimal automorphisms of $D$. Kaup, Matsushima and Ochiai [1] showed that the Lie algebra $\mathfrak{g}(D)$ has the following graded structure:

$$\mathfrak{g}(D)=\mathfrak{g}^{-2}+\mathfrak{g}^{-1}+\mathfrak{g}^0+\mathfrak{g}^1+\mathfrak{g}^2 \qquad ([\mathfrak{g}^p, \mathfrak{g}^q] \subset \mathfrak{g}^{p+q}),$$
$$\mathfrak{r}=\mathfrak{r}^{-2}+\mathfrak{r}^{-1}+\mathfrak{r}^0 \qquad (\mathfrak{r}^p=\mathfrak{r} \cap \mathfrak{g}^p),$$

where $\mathfrak{r}$ denotes the radical of $\mathfrak{g}(D)$. By using Theorem 1 we have

**Theorem 2.** *There exists a semi-simple graded subalgebra* $\mathfrak{s}=\sum_{p=-2}^{2} \mathfrak{s}^p$ *of* $\mathfrak{g}(D)$ *such that*

(1) $\mathfrak{s}^1=\mathfrak{s}^1$ *and* $\mathfrak{s}^2=\mathfrak{g}^2$,

(2) *For any* $X \in \mathfrak{s}^0$, *the condition* "$[X, \mathfrak{s}^1+\mathfrak{s}^2]=0$" *implies* $X=0$.

Let $\mathfrak{s}$ be as in Theorem 2. Since $\mathfrak{s}$ is semi-simple, there exists a unique element $E_s$ of $\mathfrak{s}^0$ such that

$$[E_s, X]=pX \qquad \text{for } X \in \mathfrak{s}^p.$$

We set

$$\mathfrak{r}_0^{-2}=\{X \in r^{-2}\,;\, [\mathfrak{s}, X]=0\},$$
$$\mathfrak{r}_s^{-2}=\{X \in r^{-2}\,;\, [E_s, X]=-X\},$$
$$\mathfrak{r}_0^{0}=\{X \in r^{0}\,;\, [\mathfrak{s}, X]=0\},$$
$$\mathfrak{r}_s^{0}=\{X \in r^{0}\,;\, [E_s, X]=X\}.$$

In the notations as above, we have the following

**Theorem 3.** *The radical* $\mathfrak{r}$ *has the following structure*:

(1) $\mathfrak{r}^{-2}=\mathfrak{r}_0^{-2}+\mathfrak{r}_s^{-2}$ (*direct sum*), $\mathfrak{r}_0^{-2} \supset [\mathfrak{r}^{-1}, \mathfrak{r}^{-1}]$,