41. The Asymptotic Distribution of the Lower Part Eigenvalues for Elliptic Operators

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1. Introduction. Let A be a positive homogeneous elliptic operator with constant coefficients defined on R^n. We consider the eigenvalue problem of the following form

(1.1) $$Au - pu = \lambda u.$$

Here $p(x)$ is a positive function with $\lim_{|x|\to\infty} p(x)=0$. If $p(x)$ does not approach to zero too rapidly at infinity, then the operator $A-p$ has an infinite sequence of negative eigenvalues approaching to zero. We denote by $n(r)$ $(r>0)$ the number of eigenvalues less than $-r$ of problem (1.1). In this note we study the asymptotic behavior of $n(r)$ as $r\to 0$. The asymptotic behavior for the Schrödinger operator with a non-smooth potential $p(x)$ was studied in Brownell and Clark [3], and McLeod [4].

Only the theorem and a sketch of its proof are presented here and the details will be published elsewhere.

2. Main result. Let $A(D)=\sum_{|\alpha|=m} a_\alpha D^\alpha$ be an elliptic operator with constant coefficients defined on R^n. We suppose that $A(\xi)\geq 0$ and denote by $K(l, a)$ $(l>0, a>0)$ the set of functions $p(x)$ which satisfy the following conditions:

(i) $p(x)$ is decomposed as $p(x)=p_1(x)+p_2(x)$;

(ii) $p_1(x)$ is a positive smooth function with $\lim_{|x|\to\infty} |x|^l p_1(x)=a$;

(iii) $p_2(x)$ is a nonnegative function with compact support;

(iv) $p_2(x)\in L_p$, where $p=1$ if $m\geq n$ and $p>n/m$ if $m<n$.

Theorem. *Let A be an elliptic operator satisfying the above conditions and suppose that $p(x)$ belongs to $K(l, a)$ and that $l<m$. Then,*

(2.1) $$n(r)=(2\pi)^{-n}\omega\frac{S}{n}a^{n/l}r^{n/m-n/l}+o(r^{n/m-n/l})$$

where $\omega=\int_{R^n}\frac{d\xi}{(A(\xi)+1)^{n/l}}$ *and S is the surface measure of the $n-1$ dimensional unit sphere if $n\geq 2$ and $S=2$ if $n=1$.*

Remark. Theorem 1 can be extended to the case that $A(D)$ is an inhomogeneous elliptic operator. The details will be discussed in the forthcoming paper.

3. Outline of the proof. In Birman [1], it was shown that $n(r)$ coincides with the number of eigenvalues μ less than 1 of the following eigenvalue problem